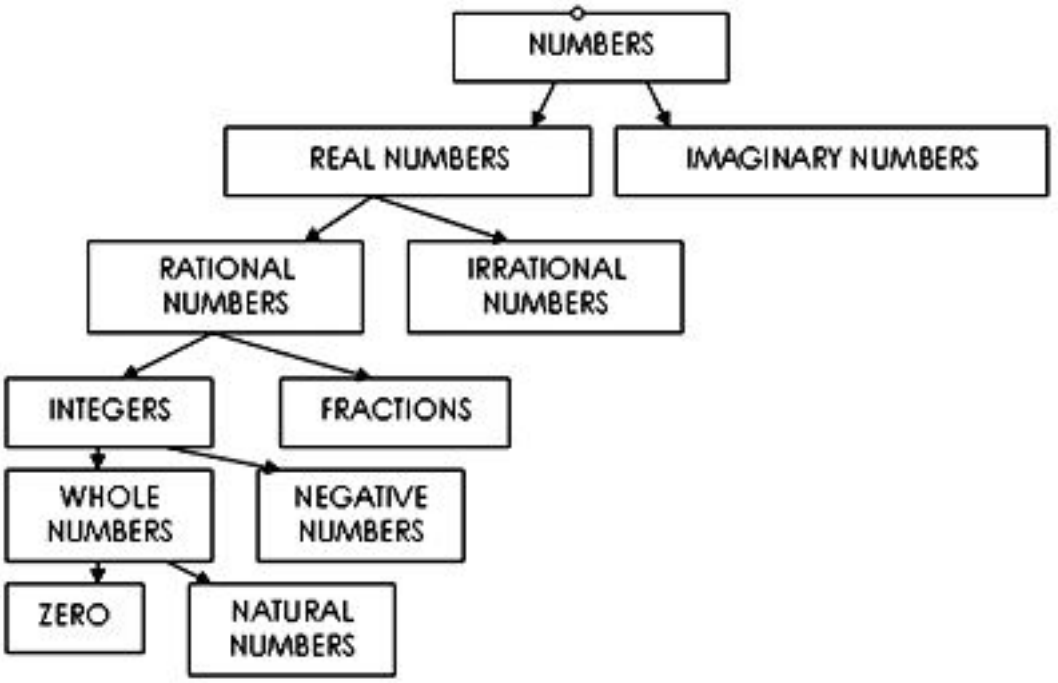
1.4 Determining the Domain & Range of a Function

Class notes and links for MCR3U Period One class

KEY TERMS

Note the numbering systems:



Real Numbers: numbers that are either rational or irrational; these include positive and negative integers, zero, fractions and irrational numbers such as $\sqrt{2}$ and π.

Set Notation: used to describe domains and ranges. For example, $\{x \varepsilon \Re \mid 0 \leq x < 50\}$ is read "the set of all values x that belong to the set of real numbers, such that x is greater than or equal to 0 and less than 50". The symbol "|" stands for "such that".

Example One:

When looking at a graph, the domain is simply the set of values for which there are existing x values. Likewise, the range is simply the set of existing f(x) values.

Once you know all the x values that exist you can write a domain statement that reads

$\{x \varepsilon \Re \mid x>6\}$

In this case, it reads "x is an element (ε) of all the real numbers ($\Re$) such that (|) x is greater than 6".

$\{x \varepsilon \aleph\}$

In this case, it reads "x is an element (ε) of all the natural numbers ($\aleph$)".

Likewise, when writing a range statement you can write:

$\{f(x) \varepsilon W | f(x) \neq 14\}$

In this case, it reads “f(x) is an element (ε) of all the whole numbers (W), such that f(x) cannot equal 14”.

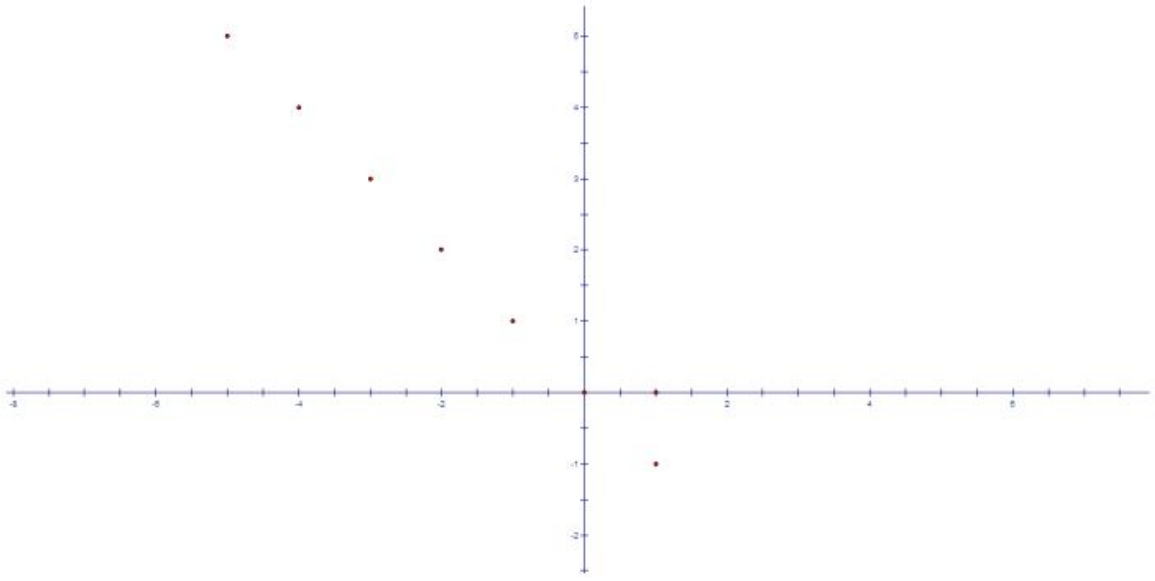
Try to read what this says:

$\{f(x) \varepsilon \Re | f(x)<0\}$

Did you say: “f(x) is an element of all the real numbers, such that f(x) is less than 0”.

Example Two:

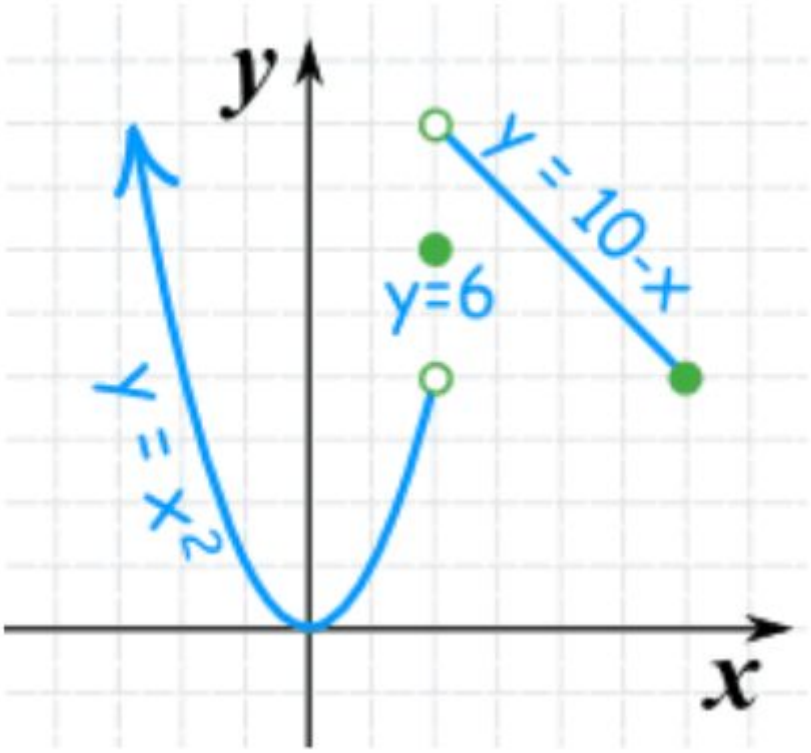
Find the domain of this function.



In this example, we only have points, so the domain statement will read $\{x \varepsilon I | -5 \leq x \leq 1\}$. The range statement will similarly read $\{f(x) \varepsilon I | -1 \leq x \leq 5\}$. This is a function.

Example Three:

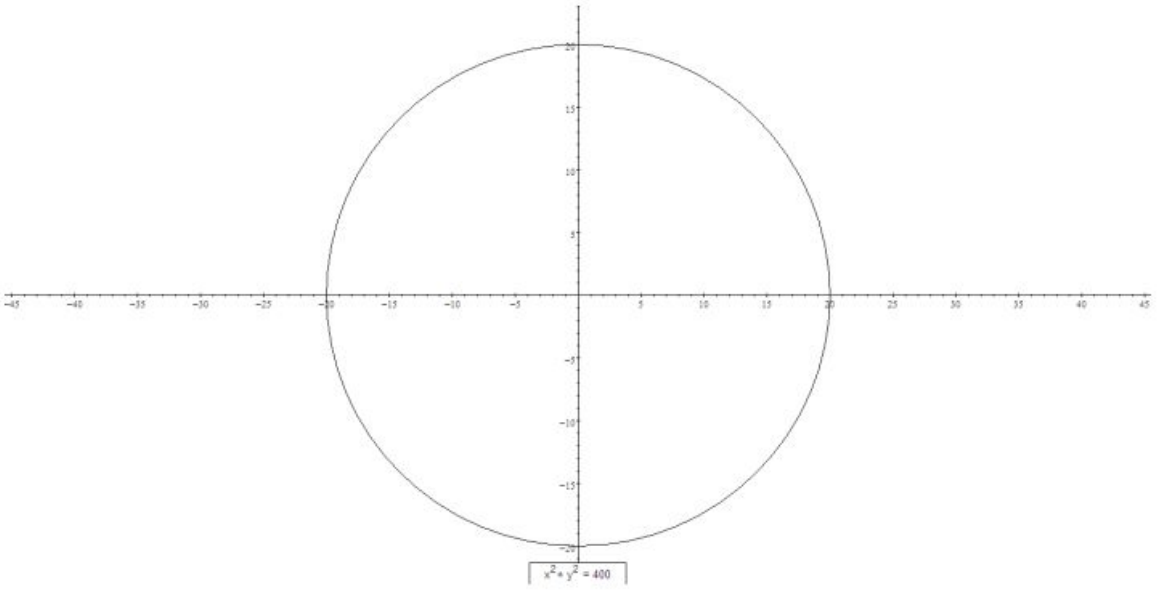
Find the domain and range. Is this a function?



In this example, we have a piecewise function. The domain statement will read $\{x \varepsilon \Re | -\infty < x \leq 6\}$ OR $\{x \varepsilon \Re | x \leq 6\}$. The range statement will read $\{f(x) \varepsilon \Re | f(x) \geq 0\}$. This is a function due to the vertical line test.

Example Four:

Find the domain and range. Is this a function?



In this example, we have a circle. This automatically does not pass the vertical line test; therefore, it is a relation but not a function. The domain statement will read {xεR|-20≤x≤20}. The range statement will read {yε$\Re$|-20≤y≤20}.

Challenge: Determine the domain and ranges from the following equations:

a) f(x) = 4x - 7

b) g(x) = $-2(x-4)^2 + 3$

c) h(x) = $(5-x)^{½}$

Solutions

a) f(x) = 4x - 7

The domain will be {xε$\Re$} and the range will be {f(x)ε$\Re$} because this is a linear function. There are no restrictions on either x or f(x).

b) g(x) = $-2(x-4)^2 + 3$

The domain will be {xε$\Re$} and the range will be {f(x)ε$\Re$|f(x)≤3} because this is a quadratic function. There are no restrictions on either x but since the vertex is at (4,3) and the parabola opens downwards, f(x)≤3.

c) $h(x) = (5-x)^{½}$

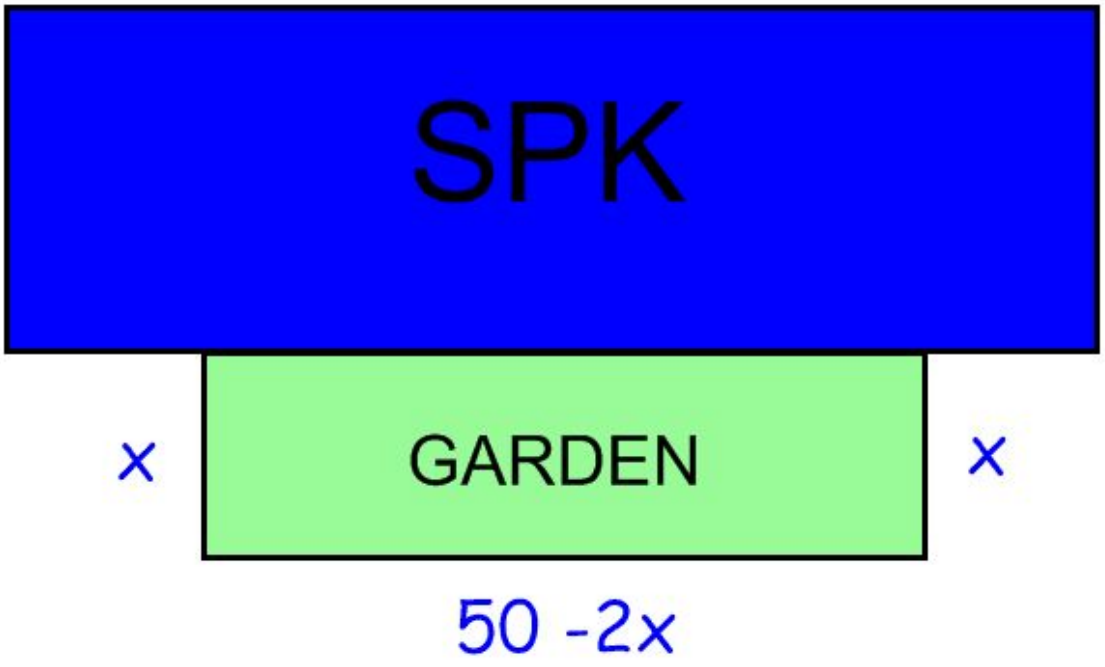
The domain will be $\{x \varepsilon \Re | x \leq 5\}$ and the range will be $\{f(x) \varepsilon \Re | f(x) \geq 0\}$ because this is a square root function. Since square roots cannot have negative input values, x cannot exceed 5. Ergo, the range values will always be either zero or positive.

The Garden Problem

The SPK gardening club is building a fence around their garden which is adjacent to one of the school's exterior walls. They have been allotted 50 m of fencing to complete this task.

a) Express the area of the garden as a function of its width.

b) Determine the domain and range of the area function.



Solution:

a) Express the area of the garden as a function of its width.

$A = lw$

$A(w) = w(50-2w)$

$A(w) = 50w - 2w^2$

b) Determine the domain and range of the area function.

Domain Statement: $\{w \varepsilon \Re | 0<w<25\}$

Range Statement: $\{A(w) \varepsilon \Re | 0<A(w) \leq 312.5\}$

There are times when theoretical mathematics needs to take a break and we must allow for reality to set in. In this case, the domain and range function must take reality in consideration despite the function that we have. The consequence would be that we would get a zero area value? or negative width values? Hence, we know that the domain must be greater than zero and the maximum width would be less than 25m; ergo the domain. Likewise, our range (the area) must be greater than zero but must be less than the optimal width. Recall that this is a parabola so the maximum will be between the two zeros (0 and 25), hence 12.5. The area with a width of 12.5 is 312.5, hence the domain.

KEY IDEAS

The domain of a function is the set of values of the independent variable for which the function is defined. The range of a function depends on the equation of the function. The graph depends on the domain and range.

The domain and range of a function can be determined from its graph, from a table of values or from the function equation. They are usually easier to determine from a graph or a table of values.

All linear functions include all the real numbers in their domain. Linear functions of the form $f(x) = mx + b$, where $m \neq 0$, have range $\{y \varepsilon \Re\}$. Constant functions $f(x) = b$ have range $\{b\}$.

All quadratic functions have domain $\{x \varepsilon \Re\}$. The range of a quadratic function depends on the maximum or minimum value and the direction of opening.

The domain of square root functions are restricted because of the square root of a negative number is not a real number (it is an imaginary number … later in your life). The ranges are restricted because the square root sign refers to the positive square root. For example,

The function $f(x) = \sqrt{x}$, has domain $D = \{x \varepsilon \Re | x \geq 0\}$ and domain $\{y \varepsilon \Re | y \geq 0\}$.

The function $g(x) = \sqrt{(x-3)}$, has domain $D = \{x \varepsilon \Re | x \geq 3\}$ and domain $\{y \varepsilon \Re | y \geq 0\}$.

When working with functions that model real-world situations, consider whether there are any restrictions on the variables. For example, negative values often have no meaning in a real context, so the domain or range must be restricted to realistic values.

HOMEWORK

- 1.4 Determining the Domain & Range of a Function: 1-3,5,9,15,16