



Functions and Applications

Chapter 3: Working with Quadratic Functions
Standard and Factored Forms

3.2 Relating the Standard and Factored Forms



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Learning Goals:

- Some quadratic functions in standard form, $f(x) = ax^2 + bx + c$ can be expressed in factored form as $f(x) = a(x - r)(x - s)$ by factoring. The two forms of the quadratic function are equivalent.
- All quadratic functions in factored form can be expressed in standard form by expanding. The two forms of the quadratic function are equivalent.



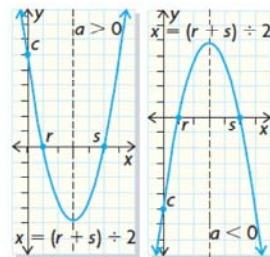
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3.2 Relating the Standard and Factored Forms

Learning Goals:

- Both the standard and factored forms provide useful information for graphing the parabola.
- When a quadratic function is expressed in factored form, $f(x) = a(x - r)(x - s)$, r and s are the x -intercepts, or zeros, of the function. The axis of symmetry is the vertical line that runs through the midpoint of the zeros and is defined by $x = (r + s) \div 2$. This value is also the x -coordinate of the vertex.



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Learning Goals:

- When a quadratic function is expressed in standard form, $f(x) = ax^2 + bx + c$, c is the y -intercept of the function.
- The value of a in both factored and standard forms determines the direction that the parabola opens. If $a > 0$, the parabola opens up; if $a < 0$, the parabola opens down.



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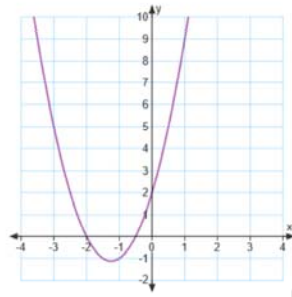
3.2 Relating the Standard and Factored Forms

Standard Form: $f(x) = ax^2 + bx + c$

What do we know?

a = Direction of opening

c = y - intercept ($x = 0$)



$$f(x) = 2x^2 + 5x + 2$$



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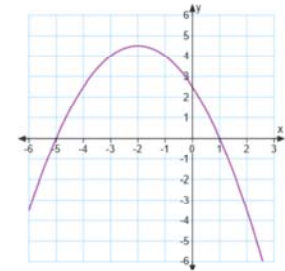
3.2 Relating the Standard and Factored Forms

Factored Form: $f(x) = a(x + r)(x + s)$

What do we know?

a = Direction of opening

c = x - intercepts ($y = 0$)



$$f(x) = 1/2(x + 5)(x - 1)$$



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3.2 Relating the Standard and Factored Forms

Converting from standard form to vertex form

Simply factor the function

Example:

$$f(x) = 3x^2 + 21x + 36$$

$$f(x) = 3(x^2 + 7x + 12)$$

$$f(x) = 3(x + 4)(x + 3)$$



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Converting from vertex form to standard form

Simply expand the function

Example:

$$g(x) = -5(x + 2)(x - 6)$$

$$g(x) = -5(x^2 + 2x - 6x - 12)$$

$$g(x) = -5(x^2 - 4x - 12)$$

$$g(x) = -5x^2 + 20x + 60$$

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Determining the vertex from factored form:

$$\text{Given } f(x) = 2(x - 2)(x + 4)$$

Step 1: Find the zeros

Either:

$$\begin{array}{l} (x - 2) = 0 \quad \text{or} \quad (x + 4) = 0 \\ x = 2 \quad \quad \quad \quad x = -4 \end{array}$$

So the co-ordinates of our zeroes are (2,0) and (-4, 0)

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Determining the vertex from factored form:

Step 2: Locate the axis of symmetry

The zeroes were (-4,0) and (2,0) and the axis of symmetry must lie exactly between those two points.

We just take the mean of the two values:

$$\begin{array}{l} \frac{(-4) + 2}{2} \quad \text{So we know that the axis of symmetry is} \\ = \frac{-2}{2} \quad \text{the line } x = -1. \\ = -1 \quad \text{This is the x co-ordinate of the vertex.} \end{array}$$

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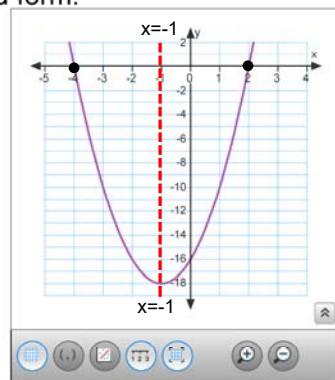
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Determining the vertex from factored form:

Step 2: Locate the axis of symmetry

$$y = 2(x - 2)(x + 4)$$

$$x = -1$$



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Determining the vertex from factored form:

Step 3: Find the y co-ordinate of the vertex.

We know that the vertex is at $x = -1$, so we insert this back into our function to find the y - value.

$$f(x) = 2(x - 2)(x + 4)$$

$$f(-1) = 2((-1) - 2)((-1) + 4)$$

$$f(-1) = 2(-3)(3)$$

$$f(-1) = -18$$

The minimum of this parabola is -18

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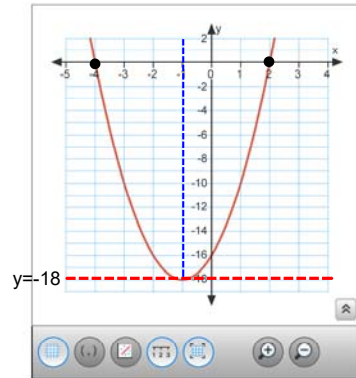
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Determining the vertex from factored form: $y = 2(x - 2)(x + 4)$

Step 3: Find the y co-ordinate of the vertex

$$y = -18$$



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Homework:

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