

§ 3.4 Operations with Radicals

Definition: A radical occurs when a number or variable is rooted! A radical can be a square root $\sqrt{5}$, or $\sqrt{x+3}$, a cubed root $\sqrt[3]{4}$, $\sqrt[3]{4x-1}$, or any n^{th} root $\sqrt[5]{4}$, $\sqrt[9]{x-3}$, $\sqrt[1215]{x+2}$.

We can also have entire radicals (as above), where there is a coefficient of 1.

We can also have a mixed radical which has a coefficient other than 1, ex: $2\sqrt{5}$, $3\sqrt{7}$.

Reflective Activity

Turn to §3.4 of your books and complete the reflective assignment A-E. You have twenty minutes.

Example: Express each of the following as a mixed radical in lowest terms.

a) $\sqrt{180}$

b) $\sqrt{98}$

Solution a):

$$\sqrt{180}$$

$$\sqrt{(36)(5)}$$

$$6\sqrt{5}$$

Solution b):

$$\sqrt{98}$$

$$\sqrt{(49)(2)}$$

$$7\sqrt{2}$$

Example: Express each of the following as entire radicals.

a) $3\sqrt{5}$

b) $-4\sqrt{13}$

Solution a):

$$3\sqrt{5}$$

$$\sqrt{9}\sqrt{5}$$

$$\sqrt{45}$$

Solution b):

$$-4\sqrt{13}$$

$$-\sqrt{16}\sqrt{13}$$

$$-\sqrt{208}$$

Example: Simplify.

a) $\sqrt{7}\sqrt{3}$

b) $-5\sqrt{5} \times 3\sqrt{5}$

Solution a):

$$\sqrt{7}\sqrt{3}$$

$$\sqrt{21}$$

Solution b):

$$-5\sqrt{5} \times 3\sqrt{5}$$

$$-15\sqrt{5}\sqrt{5}$$

$$-15\sqrt{25}$$

$$-15(5)$$

$$-75$$

Example: Test the following statements.

a) $\sqrt{7} + \sqrt{3} = \sqrt{10}$

b) $3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}$

Solution a) and b):

Left side Right side

$\sqrt{7} \approx 2.65$ $\sqrt{10} \approx 3.16$

$\sqrt{3} \approx 1.73$

$\sqrt{7} + \sqrt{3}$

$2.65 + 1.73$

4.38

$\sqrt{7} + \sqrt{3} \neq \sqrt{10}$

Left side

$3\sqrt{2} + 2\sqrt{2}$

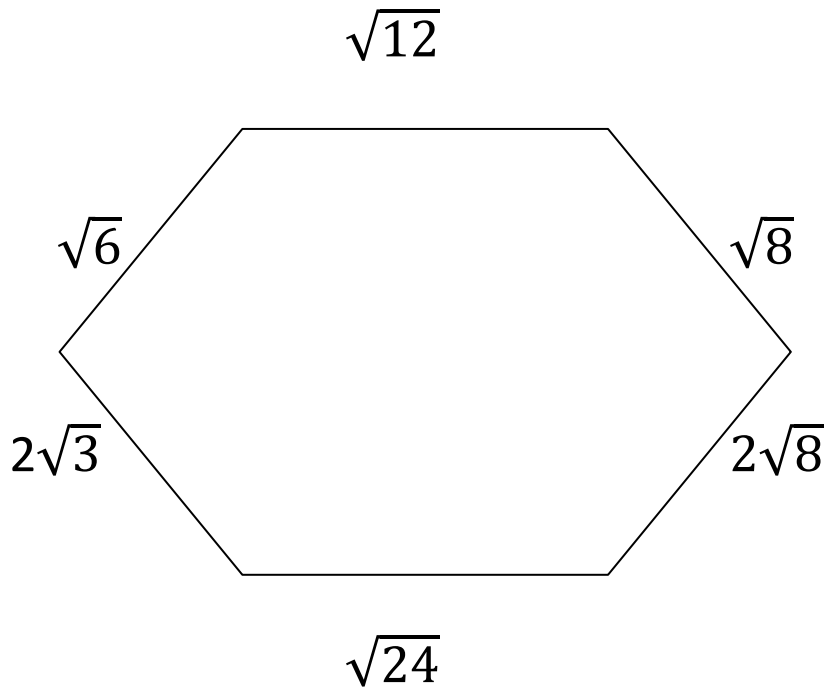
$\sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2} + \sqrt{2}$

$5\sqrt{2}$

Hence

$3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}$

Example: Find the perimeter of the following shape.



Solution:

$$P = \sqrt{12} + \sqrt{6} + \sqrt{8} + 2\sqrt{3} + 2\sqrt{8} + \sqrt{24}$$

$$P = \sqrt{4 \times 3} + \sqrt{6} + \sqrt{4 \times 2} + 2\sqrt{3} + 2\sqrt{4 \times 2} + \sqrt{4 \times 6}$$

$$P = 2\sqrt{3} + \sqrt{6} + 2\sqrt{2} + 2\sqrt{3} + 4\sqrt{2} + 2\sqrt{6}$$

$$P = 6\sqrt{2} + 4\sqrt{3} + 3\sqrt{6}$$

Example: Simplify.

$$(7-\sqrt{3})(3+\sqrt{27})$$

Solution:

$$(7-\sqrt{3})(3+\sqrt{27})$$

$$21+7\sqrt{27}-3\sqrt{3}-\sqrt{81}$$

$$(21-9)+7\sqrt{9 \times 3}-3\sqrt{3}$$

$$12+21\sqrt{3}-3\sqrt{3}$$

$$12+ 18\sqrt{3}$$

Alternate Solution:

$$(7-\sqrt{3})(3+\sqrt{27})$$

$$(7-\sqrt{3})(3+3\sqrt{3})$$

$$21+21\sqrt{3}-3\sqrt{3}-3\sqrt{9}$$

$$21+18\sqrt{3}-9$$

$$12+ 18\sqrt{3}$$