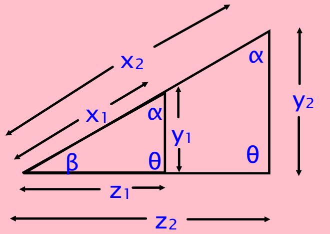
St. Peter Catholic High School
Algonquin College
MCT4C
Mr. M. Couturier
www.habfanforever.com

Jan 27-10:05 AM

§4.2 Defining the Trigonometric Functions

Let us start with a reminder that *similar* triangles all have equal corresponding angles while having proportional side lengths.



These two triangles are similar.

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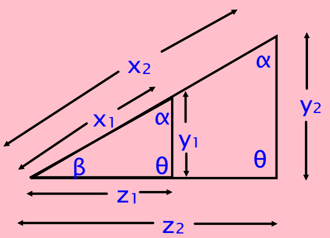
Hence since they are similar we can say:

$\frac{x_2}{x_1} = \frac{y_2}{y_1} = \frac{z_2}{z_1}$

$\alpha_1 = \alpha_2$
$\beta_1 = \beta_2$
$\theta_1 = \theta_2$



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Now let's take that application to derive the primary trigonometric ratios from the following design:

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

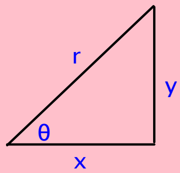
$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

$\tan\theta = \frac{y}{x}$



From this we get the famous saying:

SOH CAH TOA

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SOH CAH TOA is just a fun way of saying

sine is opposite over hypotenuse
cosine is adjacent over hypotenuse and
tangent is opposite over adjacent

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Now let's define the reciprocal trigonometric functions

$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$

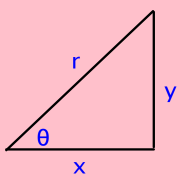
$\csc\theta = \frac{r}{y}$

$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$

$\sec\theta = \frac{r}{x}$

$\cot\theta = \frac{\text{adjacent}}{\text{opposite}}$

$\cot\theta = \frac{x}{y}$



csc is the cosecant or 1/sin
sec is the secant or 1/cos
cot is the cotangent or 1/tan

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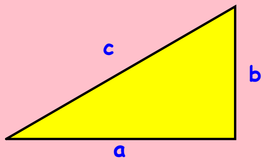
3-4-5 Pythagorean Triple

Recall the Pythagorean Theorem which states:

$$a^2 + b^2 = c^2$$

The 3-4-5 triangle is known as a Pythagorean Triple since it satisfies the Pythagorean Theorem.



Check: $a^2 + b^2 = c^2$

$(3)^2 + (4)^2 = (5)^2$

$9 + 16 = 25$

$25 = 25$

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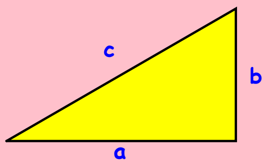
Similar Triangles

We saw that a 3-4-5 was a Pythagorean Triple, but now lets double the size of the triangle and see if it still works.

Let's test the 6-8-10



Check: $a^2 + b^2 = c^2$

$(6)^2 + (8)^2 = (10)^2$

$36 + 64 = 100$

$100 = 100$

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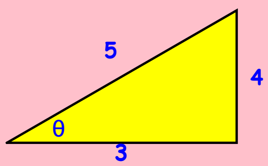
Finding angles using trigonometric functions

Let us now evaluate the angles below using the primary trigonometric functions:



$\sin\theta = 4/5$ $\csc\theta = 5/4$

$\cos\theta = 3/5$ $\sec\theta = 5/3$

$\tan\theta = 4/3$ $\cot\theta = 3/4$

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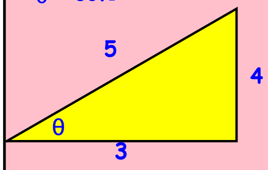
Finding angles using trigonometric functions

To evaluate the angle, take one of the trigonometric functions and solve algebraically:

$\sin \theta = 4/5$
$\theta = \sin^{-1}(4/5)$
$\theta \approx 53.1°$



Make sure that your calculator is in DEG mode!!!

No which trigonometric function you chose, you will get the same answer! Try it now!

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Example:

Find the values of all the trigonometric functions whose terminal sides passes through the point (9.23, 23.11).

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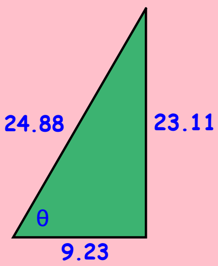
Example:

Find the values of all the trigonometric functions whose terminal sides passes through the point (9.23, 23.11).

First let us draw the triangle.

Now let us find the hypotenuse:

$a^2 + b^2 = c^2$
$c^2 = (9.23)^2 + (23.11)^2$
$c^2 = 85.19 + 534.07$
$c^2 = 619.26$
$c \approx 24.88$



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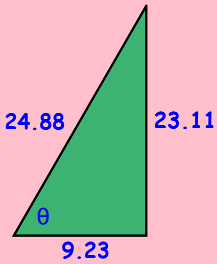
Example:

Now we have all the information we need:

$\sin\theta = 23.11/24.88 = 0.9289$
$\cos\theta = 9.23/24.88 = 0.3709$
$\tan\theta = 23.11/9.23 = 2.5038$
$\csc\theta = 24.88/23.11 = 1.0766$
$\sec\theta = 24.88/9.23 = 2.6956$
$\cot\theta = 9.23/23.11 = 0.3994$



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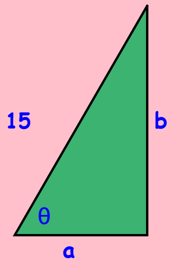
Finding a length using Trigonometric Functions

Let's say that you were told that the hypotenuse was 15m long and that $\sin\theta = 1/4$. What can you deduce from this information?



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Example: Given that h = 15m and $\sin\theta = 1/4$.

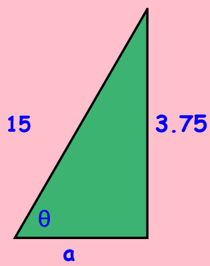
We know that:

$\sin\theta = 1/4$ and $\sin\theta = b/15$ hence

$$\frac{b}{15} = \frac{1}{4}$$

Solve for b to get:
b = 3.75

Now we can use the Pythagorean Theorem to get a!



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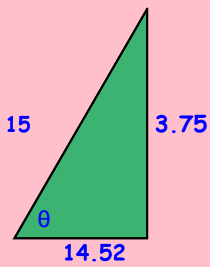
$a^2 + b^2 = c^2$
$a^2 = c^2 - b^2$
$a^2 = (15)^2 - (3.75)^2$
$a^2 = 225 - 14.06$
$a^2 = 210.94$
$a \approx 14.52$
Hence
$\sin \theta = 3.75/15 = 0.25$
$\cos \theta = 14.52/15 = 0.986$
$\tan \theta = 3.75/14.52 = 0.258$
$\csc \theta = 15/3.75 = 4$
$\sec \theta = 15/14.52 = 1.033$
$\cot \theta = 14.52/3.75 = 3.872$



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Exit Ticket: In class/group activity:
Complete & hand-In. §4.2 #16&24

Homework:
§4.2 #3-27 (odd only)

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