Functions and Applications

Chapter 4: Working with Quadratic Models - Standard and Vertex Forms

4.2 Relating the Standard and Vertex Forms : Completing the Square

Functions and Applications

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4.2 Relating the Standard and Vertex Forms : Completing the Square

Learning Goals:

- All quadratic functions in standard form can be written in vertex form by completing the square. The equations are equivalent.
- Both the standard form and the vertex form provide useful information for graphing the parabola.
- Any quadratic function in standard form with $a \neq 1$ can be expressed in vertex form when fractions are used to complete the square. Decimals can be used only if the coefficient of x results in a terminating decimal when a is factored from the x^2- and x-terms of the quadratic.

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Learning Goals:

- To complete the square, follow these steps:

$f(x) = ax^2 + bx + c$

$f(x) = a\left(x^2 + \frac{b}{a}x\right) + c$ ← Factor the coefficient of x^2 from the first two terms.

$f(x) = a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right] + c$ ← Add and subtract the square of half the coefficient of x inside the brackets.

$f(x) = a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right] - a\left(\frac{b}{2a}\right)^2 + c$ ← Group the three terms that form the perfect square. Multiply the fourth term by a, and move it outside the brackets.

$f(x) = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$ ← Factor the perfect square and simplify.

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Completing the Square

Complete the square to re-write the following in vertex form:

$y = x^2 - 10x$

$y = x^2 - 10x + (5)^2 - (5)^2$

$y = x^2 - 10x + 25 - 25$

$y = (x^2 - 10x + 25) - 25$

$y = (x - 5)^2 - 25$

We need to add and subtract a value to complete the square. We need a value that is half the 'b' term, and then we square it.

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Completing the Square

Complete the square to re-write the following in vertex form:

$$y = x^2 + 14x$$

$$y = x^2 + 14x + (7)^2 - (7)^2$$

$$y = x^2 + 14x + 49 - 49$$

$$y = (x^2 + 14x + 49) - 49$$

$$y = (x-7)^2 - 49$$

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Completing the Square

Complete the square to re-write the following in vertex form:

$$y = x^2 - 20x + 35$$

$$y = x^2 - 20x + (10)^2 - (10)^2 + 35$$

$$y = x^2 - 20x + 100 - 100 + 35$$

$$y = (x^2 - 20x + 100) - 100 + 35$$

$$y = (x-10)^2 - 65$$

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Completing the Square

Complete the square to re-write the following in vertex form:

$$y = -2x^2 - 12x + 17$$

$$y = -2\left[x^2 + 6x\right] + 17$$

$$y = -2\left[x^2 + 6x + (3)^2 - (3)^2\right] + 17$$

$$y = -2\left[x^2 + 6x + 9 - 9\right] + 17$$

$$y = -2\left[x^2 + 6x + 9\right](-2)(-9) + 17$$

$$y = -2\left[x+3\right]^2 + 18 + 17$$

$$y = -2\left[x+3\right]^2 + 35$$

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Completing the Square

Complete the square to re-write the following in vertex form:

$$y = 3x^2 - 15x + 8$$

$$y = 3\left[x^2 - 5x\right] + 8$$

$$y = 3\left[x^2 - 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2\right] + 8$$

$$y = 3\left[x^2 - 5x + \frac{25}{4} - \frac{25}{4}\right] + 8$$

$$y = 3\left[x^2 - 5x + \frac{25}{4}\right] + (3)\left(\frac{-25}{4}\right) + 8$$

$$y = 3\left[x - \frac{5}{2}\right]^2 - \frac{75}{4} + 8$$

$$y = 3\left[x - \frac{5}{2}\right]^2 - \frac{75}{4} + \frac{32}{4}$$

$$y = 3\left[x - \frac{5}{2}\right]^2 - \frac{43}{4}$$

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Completing the Square

Complete the square to re-write the following in vertex form:

$$y = -5x^2 + 45x - 29$$

$$y = -5\left[x^2 - 9x\right] - 29$$

$$y = -5\left[x^2 - 9x + \left(\frac{9}{2}\right)^2 - \left(\frac{9}{2}\right)^2\right] - 29$$

$$y = -5\left[x^2 - 9x + \frac{81}{4} - \frac{81}{4}\right] - 29$$

$$y = -5\left[x^2 - 9x + \frac{81}{4}\right](-5)\left(\frac{-81}{4}\right) - 29$$

$$y = -5\left[x - \frac{9}{2}\right]^2 + \frac{405}{4} - 29$$

$$y = -5\left[x - \frac{5}{2}\right]^2 + \frac{405}{4} - \frac{156}{4}$$

$$y = -5\left[x - \frac{5}{2}\right]^2 + \frac{249}{4}$$

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Homework: Day ONE

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Homework: Day ONE

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