# § 4.2 Working with Integer Exponents

Recall from grade 9, the exponent laws:

The Exponent Law of Multiplication states:

$$(b^m)(b^n) = b^{m+n}$$

The Exponent Law of Division states:

$$\frac{b^m}{b^n} = b^{m-n}$$

The Power Law states:

$$(b^m)^n = b^{mn}$$

The metric system is based on a base 10 system.

| Name | Symbol | Multiple of the metre | Multiple as a power of 10. |
|---|---|---|---|
| terametre | Tm | 1,000,000,000,000 | $10^{12}$ |
| gigametre | Gm | 1,000,000,000 | $10^{9}$ |
| megametre | Mm | 1,000,000 | $10^{6}$ |
| kilometre | km | 1,000 | $10^{3}$ |
| hectometre | hm | 100 | $10^{2}$ |
| decametre | dam | 10 | $10^{1}$ |
| metre | m | 1 | $10^{0}$ |
| decimetre | dm | 0.1 | $10^{-1}$ |
| centimetre | cm | 0.01 | $10^{-2}$ |
| millimetre | mm | 0.001 | $10^{-3}$ |
| micrometre | μm | 0.000 001 | $10^{-6}$ |
| nanometre | nm | 0.000 000 001 | $10^{-9}$ |
| picometre | pm | 0.000 000 000 001 | $10^{-12}$ |
| femtometre | fm | 0.000 000 000 000 001 | $10^{-15}$ |
| attometre | am | 0.000 000 000 000 000 001 | $10^{-18}$ |

What is $a^x$ if x=0?

A lot of people struggle with the idea that $a^0 = 1$.

Why?

Let's look at two ways to demonstrate this idea.

First let's look at:

$$\frac{a^5}{a^5} = 1$$

We know that this is equal to 1. However, we also know:

$$\frac{(a)(a)(a)(a)(a)}{(a)(a)(a)(a)(a)} = 1$$

We can also say that:

$$\frac{a^5}{a^5} = a^{5-5}$$

$$\frac{a^5}{a^5} = a^0$$

$$a^0 = 1$$

We have seen that a negative exponent DOES NOT yield a negative value, but rather the RECIPROCAL of itself.

Example #1: Express $a^{-n}$ as a positive exponent.

Solution:

$$a^{-n} = \frac{1}{a^n}$$

Example #2: Express $(\frac{a}{b})^{-n}$ as a positive exponent.

Solution:

$$(\frac{a}{b})^{-n} = (\frac{1}{\frac{a}{b}})^{n}$$

$$(\frac{a}{b})^{-n} = (\frac{b}{a})^{n}$$

Where $a \neq 0$, $b \neq 0$.

## Evaluate:

a) $3^{-4}$ b) $(-5)^{-3}$ c) $-7^{-2}$

## Solution:

a) $3^{-4}$ b) $(-5)^{-3}$ c) $-7^{-2}$

$3^{-4} = \frac{1}{3^4}$ $(-5)^{-3} = \frac{1}{(-5)^3}$ $-7^{-2} = -\frac{1}{7^2}$

$3^{-4} = \frac{1}{81}$ $(-5)^{-3} = -\frac{1}{125}$ $-7^{-2} = -\frac{1}{49}$

**Evaluate:** $\left(\frac{3}{4}\right)^{-3}$

**Solution:**

$$\left(\frac{3}{4}\right)^{-3} = \left(\frac{1}{\frac{3}{4}}\right)^{3}$$

$$\left(\frac{3}{4}\right)^{-3} = \left(\frac{1}{\frac{27}{64}}\right)$$

$$\left(\frac{3}{4}\right)^{-3} = \frac{64}{27}$$

**Evaluate:** $\frac{(7^2)\,(7^{-3})}{7(7^{-5})^2}$

**Solution:**

$$\frac{(7^2)\,(7^{-3})}{7(7^{-5})^2} = \frac{(7^2)}{7(7^3)(7^{-10})}$$

$$\frac{(7^2)\,(7^{-3})}{7(7^{-5})^2} = \frac{(7^{10})(7^2)}{7^4}$$

$$\frac{(7^2)\,(7^{-3})}{7(7^{-5})^2} = \frac{(7^{12})}{7^4}$$

$$\frac{(7^2)\,(7^{-3})}{7(7^{-5})^2} = 7^8$$

$$\frac{(7^2)\,(7^{-3})}{7(7^{-5})^2} = 5{,}764{,}801$$

**Homework: §4.2#1-3,4cd,5cd,6cd,7def,8cd, 9cd,11ad,13ceg,14e,15**