Functions and Applications

Chapter 4: Working with Quadratic Models - Standard and Vertex Forms

4.3 Solving Quadratic Equations Using the Quadratic Formula

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4.3 Solving Quadratic Equations Using the Quadratic Formula

Learning Goals

- All quadratic equations of the form $ax^2 + bx + c = 0$ can be solved using the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- The quadratic formula is derived by completing the square for $ax^2 + bx + c = 0$ and solving for x. It is a direct way of calculating roots without graphing or algebraic manipulation.
- A quadratic equation can have 2, 1, or 0 real solutions, depending on the values of a, b, and c.
- The solutions generated by the quadratic formula for the equation $ax^2 + bx + c = 0$ correspond to the zeros, or x-intercepts, of the function $f(x) = ax^2 + bx + c$.

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The Quadratic Formula:

The roots of any quadratic of the form:

$f(x)=ax^2+bx+c$

can be found using:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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4.3 Solving Quadratic Equations Using the Quadratic Formula

How to derive the quadratic formula:

$$0 = ax^2 + bx + c$$

$$0 = a\left(x^2 + bx\right) + c$$

$$0 = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) + c$$

$$0 = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c$$

$$0 = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - a\frac{b^2}{4a^2} + c$$

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How to derive the quadratic formula: (continued, page 2)

$$0 = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - a\frac{b^2}{4a^2} + c$$

$$0 = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a} + c\frac{4a}{4a}$$

$$0 = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + \frac{4ac}{4a}$$

$$0 = a\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2}{4a} - \frac{4ac}{4a}\right)$$

$$0 = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a}$$

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How to derive the quadratic formula: (continued, page 3)

$$0 = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a}$$

$$\frac{b^2 - 4ac}{4a} = a\left(x + \frac{b}{2a}\right)^2$$

$$\frac{b^2 - 4ac}{4a} \bullet \frac{1}{a} = \left(x + \frac{b}{2a}\right)^2$$

$$\frac{b^2 - 4ac}{4a^2} = \left(x + \frac{b}{2a}\right)^2$$

$$\pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = x + \frac{b}{2a}$$

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How to derive the quadratic formula: (continued, page 4)

$$\pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = x + \frac{b}{2a}$$

$$x = -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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4.3 Solving Quadratic Equations Using the Quadratic Formula

Use quadratic formula to solve an equation we already know:

$f(x) = x^2 - 7x + 12$ solutions: x= 3, 4

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(1)(12)}}{2(1)}$$

$$x = \frac{7 \pm \sqrt{49 - 48}}{2}$$

$$x = \frac{7 \pm \sqrt{1}}{2}$$

$$x = \frac{7 \pm 1}{2}$$

$$x = \frac{7+1}{2} \qquad x = \frac{7-1}{2}$$

$$x = \frac{8}{2} \qquad x = \frac{6}{2}$$

$$x = 4 \qquad x = 3$$

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4.3 Solving Quadratic Equations Using the Quadratic Formula

Use quadratic formula to solve an equation we already know:

$f(x) = 6x^2 - 7x + 2$ solutions: x= 1/2, 2/3

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(6)(2)}}{2(6)}$$

$$x = \frac{7 \pm \sqrt{49 - 48}}{12}$$

$$x = \frac{7 \pm \sqrt{1}}{12}$$

$$x = \frac{7 \pm 1}{12}$$

$$x = \frac{7+1}{12} \qquad x = \frac{7-1}{12}$$

$$x = \frac{8}{12} \qquad x = \frac{6}{12}$$

$$x = \frac{2}{3} \qquad x = \frac{1}{2}$$

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4.3 Solving Quadratic Equations Using the Quadratic Formula

Use quadratic formula to solve an equation we already know:

$f(x) = -5x^2 + 25x + 30$ solutions: x= -1, 6

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-25 \pm \sqrt{(25)^2 - 4(-5)(30)}}{2(-5)}$$

$$x = \frac{-25 \pm \sqrt{625 + 600}}{-10}$$

$$x = \frac{-25 \pm \sqrt{1225}}{-10}$$

$$x = \frac{-25 \pm 35}{-10}$$

$$x = \frac{-25+35}{-10} \qquad x = \frac{-25-35}{-10}$$

$$x = \frac{+10}{-10} \qquad x = \frac{-60}{-10}$$

$$x = -1 \qquad x = 6$$

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4.3 Solving Quadratic Equations Using the Quadratic Formula

$f(x) = -6x^2 + 7x + 21$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(-6)(21)}}{2(-6)}$$

$$x = \frac{7 \pm \sqrt{49 + 504}}{-12}$$

$$x = \frac{7 \pm \sqrt{553}}{-12}$$

$$x = \frac{7 \pm \sqrt{553}}{-12}$$

$$x = \frac{7 + \sqrt{553}}{-12} \qquad x = \frac{7 - \sqrt{553}}{-12}$$

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Homework:

Page 222 - Questions 1-3,5,7-10,13

Mid Chapter Review:

Page 226 Question 1-11