§ 4.3 Working with Rational Exponents

Example: Find the dimensions of a surface area versus volume given a cube.

The surface area of a cube is:

$A = x^2$

We now have to solve for x, hence,

$x^2 = A$

$\sqrt{x^2} = \sqrt{A}$

$x = \sqrt{A}$

To put this in perspective, note that x is to the power of 1. How did this actually happen, when x was initially squared? Let's review this step without the square root:

$x^2 = A$

$(x^2)^n = (A)^n$ we take each side to the power of n.

What value of n makes this statement true?

$2n = 1$

$\frac{2n}{2} = \frac{1}{2}$

$n = \frac{1}{2}$

Hence, when you square root,

$(x^2)^{\frac{1}{2}} = (A)^{\frac{1}{2}}$

$x = (A)^{\frac{1}{2}}$

$x = \sqrt{A}$

The volume of a cube is:

$V = x^3$

We now have to solve for x, hence,

$x^3 = V$

How can we solve for x? We need to make the x's power 1.

$x^3 = V$

$(x^3)^n = (V)^n$ we take each side to the power of n.

What value of n makes this statement true?

$3n = 1$

$\frac{3n}{3} = \frac{1}{3}$

$n = \frac{1}{3}$

Hence, when you cube root,

$(x^3)^{\frac{1}{3}} = (V)^{\frac{1}{3}}$

$x = (V)^{\frac{1}{3}}$

$x = \sqrt[3]{V}$

What's the big idea?

If given $b^{\frac{1}{n}}$ then we can also say that b is taken to the n^{th} root or $\sqrt[n]{b}$.

As a result, if given $b^{\frac{m}{n}}$ then we can say $(\sqrt[n]{b})^m$.

Evaluate:

a) $(64)^{\frac{-1}{3}}$ b) $(196)^{\frac{1}{2}}$ c) $(10{,}000)^{\frac{1}{4}}$

Solution:

a) $(64)^{\frac{-1}{3}}$

$(64)^{\frac{-1}{3}} = \frac{1}{(64)^{\frac{1}{3}}}$

$(64)^{\frac{-1}{3}} = \frac{1}{\sqrt[3]{64}}$

$(64)^{\frac{-1}{3}} = \frac{1}{4}$

b) $(196)^{\frac{1}{2}}$

$(196)^{\frac{1}{2}} = \sqrt{196}$

$(196)^{\frac{1}{2}} = \pm 14$

c) $(10{,}000)^{\frac{1}{4}}$

$\sqrt[4]{10{,}000}$

10

Evaluate: $(27)^{\frac{2}{3}}$

Solution:

$$(27)^{\frac{2}{3}} = (\sqrt[3]{27})^2$$

$$(27)^{\frac{2}{3}} = (3)^2$$

$$(27)^{\frac{2}{3}} = 9$$

Evaluate: $(-27)^{\frac{4}{3}}$

Solution:

$$(-27)^{\frac{4}{3}} = (\sqrt[3]{-27})^4$$

$$(-27)^{\frac{4}{3}} = (-3)^4$$

$$(-27)^{\frac{4}{3}} = 81$$

Note that only odd roots can have negative bases.

Evaluate: $(16)^{-0.75}$

Solution:

$$(16)^{-0.75} = (16)^{\frac{-3}{4}}$$

$$(16)^{-0.75} = \frac{1}{16^{\frac{3}{4}}}$$

$$(16)^{-0.75} = \frac{1}{\sqrt[4]{16}^{3}}$$

$$(16)^{-0.75} = \frac{1}{2^{3}}$$

$$(16)^{-0.75} = \frac{1}{8}$$

Simplify & Evaluate: $\dfrac{8^{\frac{5}{6}}\sqrt{8}}{8^{\frac{5}{3}}}$

Solution:

$$\frac{8^{\frac{5}{6}}\sqrt{8}}{8^{\frac{5}{3}}} = \frac{8^{\frac{5}{6}}8^{\frac{1}{2}}}{8^{\frac{5}{3}}}$$

$$\frac{8^{\frac{5}{6}}\sqrt{8}}{8^{\frac{5}{3}}} = 8^{\frac{5}{6}+\frac{1}{2}-\frac{5}{3}}$$

$$\frac{8^{\frac{5}{6}}\sqrt{8}}{8^{\frac{5}{3}}} = 8^{\frac{5}{6}+\frac{3}{6}-\frac{10}{6}}$$

$$\frac{8^{\frac{5}{6}}\sqrt{8}}{8^{\frac{5}{3}}} = 8^{\frac{-2}{6}}$$

$$\frac{8^{\frac{5}{6}}\sqrt{8}}{8^{\frac{5}{3}}} = 8^{\frac{-1}{3}}$$

$$\frac{8^{\frac{5}{6}}\sqrt{8}}{8^{\frac{5}{3}}} = \frac{1}{\sqrt[3]{8}}$$

$$\frac{8^{\frac{5}{6}}\sqrt{8}}{8^{\frac{5}{3}}} = \frac{1}{2}$$

Homework:§4.3#1-4,5ef,6ef,7,8,

10,15,17,18a