St. Peter Catholic High School
Algonquin College
MCT4C
Mr. M. Couturier
www.habfanforever.com

Jan 27-10:05 AM

§8.2 Trigonometric Functions of Any Angle Part I

Imagine a law enforcement officer placing a chalk mark on your tire. When you get back into the car and drive, the mark will continuously return to that location for as long as you role.

A coterminal angle is a multiple of the original angle.

For example: if the angle was originally 60°, the next time it will return to that location will be 420° = (60°+360°).. and so on. Find another coterminal angle to 60°.

Jan 27-10:04 AM

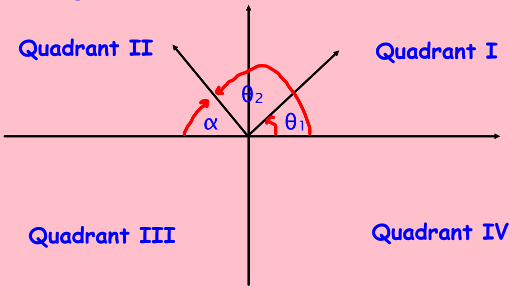
If you said: 780° = (60°+720°) or -300° = (60°-360°), then good job!

Definition: The reference angle α, is the acute angle formed by the terminal side of the angle and the x-axis.

We will now relate the reference angle to the other quadrants.

Jan 27-10:08 AM

In Quadrant II, the angle α is equal to θ_1.
Hence, $F(\theta_2) = \pm F(180° - \theta_2) = \pm F(\alpha)$, where F is the trigonometric function.



Jan 27-10:48 AM

__Example:__ Let us find the related acute angle and the trigonometric ratios given sin(129°) in Quadrant II:

Feb 16-10:34 AM

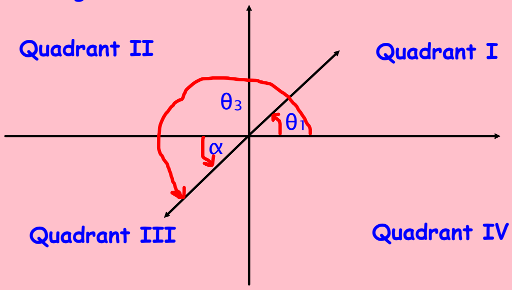
__Solution:__

$\sin(129°) = \sin(180° - 129°)$
$\sin(129°) = \sin(51°)$

$\sin(129°) = \sin(51°) = 0.777146$
$\cos(129°) = -\cos(51°) = -0.62932$
$\tan(129°) = -\tan(51°) = -1.2349$
$\sec(129°) = -\sec(51°) = -1.58907$
$\csc(129°) = \csc(51°) = 1.28676$
$\cot(129°) = -\cot(51°) = -0.809782$

Feb 16-10:38 AM

In Quadrant III, the angle α is equal to θ_1. Hence, $F(\theta_3) = \pm F(\theta_3 - 180°) = \pm F(\alpha)$, where F is the trigonometric function.



Jan 27-10:45 AM

Example: Let us find the related acute angle and the trigonometric ratios given sin(210°) in Quadrant III:

Feb 16-10:47 AM

Solution:

$\sin(210°) = -\sin(210° - 180°)$

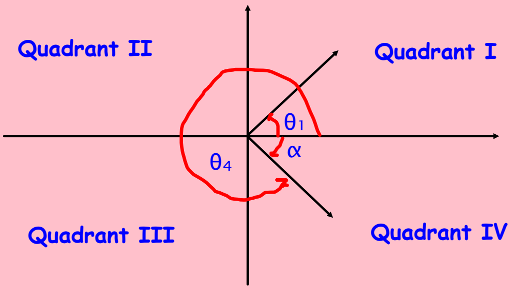
$\sin(210°) = -\sin(30°) = -0.5$
$\csc(210°) = -\csc(30°) = -2$

$\cos(210°) = -\cos(30°) = -0.866$
$\sec(210°) = -\sec(30°) = -1.155$

$\tan(210°) = \tan(30°) = 0.5774$
$\cot(210°) = \cot(30°) = 1.732$

Feb 16-10:47 AM

In Quadrant IV, the angle α is equal to θ_1. Hence, $F(\theta_4) = \pm F(360° - \theta_4) = \pm F(\alpha)$, where F is the trigonometric function.



Jan 27-10:48 AM

Example: Let us find the related acute angle and the trigonometric ratios given sin(315°) in Quadrant IV:

Feb 16-10:55 AM

Solution:

$\sin(315°) = -\sin(360° - 315°)$
$\sin(315°) = -\sin(45°) = -0.7071$
$\csc(315°) = -\csc(45°) = -1.414$

$\cos(315°) = \cos(45°) = 0.7071$
$\sec(315°) = \sec(45°) = -1.414$

$\tan(315°) = -\tan(45°) = -1$
$\cot(315°) = -\cot(45°) = -1$

Feb 16-10:55 AM

END OF PART ONE:

Exit Ticket: Complete & hand-In. §8.2 #10,16 and 26.

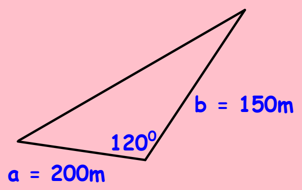
Homework:

§8.2 #5-26 all

Feb 1-12:17 PM

§8.2 Trigonometric Functions of Any Angle Part II

Example: The formula for finding the area of a triangle knowing sides a and b as well as angle C is $A=\frac{1}{2}ab\sin C$.



Jan 27-11:51 AM

Solution:

$A=\frac{1}{2}ab\sin C$

$A=\frac{1}{2}(200)(150)\sin(120)$

$A=15000\sin(120)$

$A=15000\sin(120)$

$A = 12990\ m^2$

Jan 27-11:11 AM

Calculator Note:

When a value of a trigonometric function is entered into a calculator, it is programmed to give the angle as follows:

For values of sinθ, the calculator displays angles from -90° to 90°:
{sin(90°) = -1, sin(0°) = 0, sin(90°)= 1}

For values of cosθ, the calculator displays angles from 0° to 180°:
{cos(0°) = 1, cos(90°) = 0, cos(180°)= -1}

For values of tanθ, the calculator displays angles between -90° to 90°:
{tan(0°) = 0, if tanθ<0, θ is between -90° and 0°}

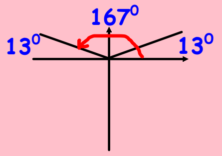
Jan 27-11:12 AM

Example: Given that sinθ = 0.2250, find the two angles for which this can happen?

Feb 1-10:02 AM

Solution:
Calculator response:
$\theta = \sin^{-1}(0.2250)$
$\theta = 13^{o}$
Now you must recall that this ratio will also be true in quadrant II, hence:
$\sin(13^{o}) = \sin(180^{o}-13^{o})$
$\sin(13^{o}) = \sin(167^{o})$

Hence we have two solutions
$\theta = 13^{o}$ and $\theta = 167^{o}$.



Feb 6-9:07 AM

Example: Given $\sec\theta = -2.722$, find all solutions for $0^o < \theta < 360^o$.

Feb 1-10:04 AM

Solution: Since
$\sec\theta = -2.722$
$\cos\theta = 1/\sec\theta$
$\theta = \cos^{-1}(1/-2.722)$
$\theta = \cos^{-1}(1/-2.722)$
$\theta = 111.6°$
Now that solution is in quadrant II. The other solution must be in quadrant III and the calculator will not reveal that answer so let us find the related acute angle:
$\alpha = 180° - 111.6°$
$\alpha = 68.5°$
Hence
$\theta = 180° + 68.5°$
$\theta = 248.5°$
Hence the two solutions are $\theta = 111.6°$ and $\theta = 248.5°$.

Feb 1-10:07 AM

Example: Given $\tan\theta = 2.050$ and $\cos\theta < 0$, find θ for $0^o < \theta < 360^o$.

Feb 16-10:28 PM

Solution: From $\tan\theta = 2.050$ we know that this can happen in quadrant I and III. Since we know that $\cos\theta<0$, we know that this can happen in quadrant II and III. Hence the solution we seek is in quadrant III.

$\tan\theta = 2.050$

$\theta = \tan^{-1}(2.050)$

$\theta = 64°$

Problem, this solution is in quadrant I. We need quadrant III, hence,

$\theta = 180°+64°$

$\theta = 244°$

Feb 16-10:31 PM

Example: What if $\tan\theta = -2.050$ and $\cos\theta<0$. Now find θ for $0^o<\theta<360^o$.

Feb 17-8:38 AM

Solution: From $\tan\theta = -2.050$ we know that this can happen in quadrant II and IV. Since we know that $\cos\theta<0$, we know that this can happen in quadrant II and III. Hence the solution we seek is in quadrant II.

$\tan\theta = -2.050$

$\theta = \tan^{-1}(-2.050)$

$\theta = -64°$

Problem, this solution is in quadrant IV. To correct it we take the absolute value of the angle, hence 64°. Now we need to find the quadrant II solution, hence,

$\theta = 180°-64°$

$\theta = 116°$

Feb 17-8:38 AM

In a nut shell:

$\theta = \alpha$	quadrant I
$\theta = 180° - \alpha$	quadrant II
$\theta = 180° + \alpha$	quadrant III
$\theta = 360° - \alpha$	quadrant IV

Feb 16-10:34 PM

Negative angles:

$\sin(-\theta) = -\sin(\theta)$ $\csc(-\theta) = -\csc(\theta)$

$\cos(-\theta) = \cos(\theta)$ $\sec(-\theta) = \sec(\theta)$

$\tan(-\theta) = -\tan(\theta)$ $\cot(-\theta) = -\cot(\theta)$

Feb 16-10:38 PM

Example:

$\sin(-45) =$ $\csc(-45) =$

$\cos(-45) =$ $\sec(-45) =$

$\tan(-45) =$ $\cot(-45) =$

Feb 16-10:44 PM

Solution:

sin(-45) = -sin(45)   csc(-45) = -csc(45)

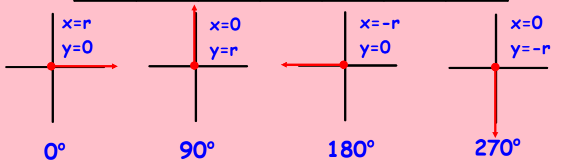
cos(-45) = cos(45)   sec(-45) = sec(45)

tan(-45) = -tan(45)   cot(-45) = -cot(45)

Feb 16-10:46 PM

Definition: A quadrantal angle is when the terminal arm is located on one of the axis.

θ	sinθ	cosθ	tanθ	cotθ	secθ	cscθ
0	0	1	0	∞	1	∞
90	1	0	∞	0	∞	1
180	0	-1	0	∞	-1	∞
270	-1	0	∞	0	∞	-1
360	0	1	0	∞	1	∞



Feb 17-8:43 AM

Exit Ticket: Complete & hand-In. §8.2 #54

Homework:

§8.2 #27-47 (odd only)

Feb 1-12:10 PM