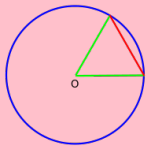
8.3 Radian Measure

Up until now you have utilized degrees in your discussion of geometry and trigonometry. The earliest believed use of the degree system of 360 degrees stems from Ancient Persia whose calendars had 360 days per year.

Nov 9-3:17 PM

The Babylonians argued that you can divide a circle into six equilateral triangles forming six chords; since all equilateral triangles have angles of 60 degrees. The Greeks later adopted these theories when they conquered the area.



Nov 9-3:32 PM

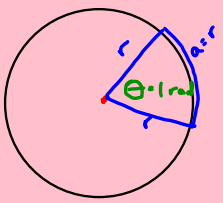
Regardless of all these theories, there is a beauty to the number 360. It is a rare number as it is so easily divisible (24 divisors and all numbers from 1 to 10 except 7).

{1,2,3,4,5,6,8,9,10,12,15,18,20, 24,30,36,40, 45,60,72,90,120,180,360}

All this to say, we need a better system.

Nov 9-3:36 PM

Let us define the following circle of radius "r" and arc length "a" subtended by the angle θ (theta). We define 1 radian to be the circle where the angle θ is subtended by an arc length equal to the radius.



Nov 13-9:47 AM

Θ is therefore unitless since a and r are of the same length units such that:

$\theta = \frac{a}{r}$

Now how many degrees are there in 1 rad?

Let us work with a well known circle where the radius is r, the arc length is the circumference (or $2\pi r$) and is subtended by 360°.

Nov 13-9:57 AM

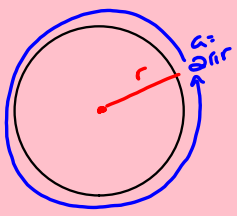
Now how many degrees are there in 1 rad?

$\theta = \frac{a}{r}$

$\theta = \frac{2\pi r}{r}$

$\theta = 2\pi$ radians

$\theta = 360^0$ or 360 degrees



Nov 9-3:01 PM

With this knowledge, let us create a Rosetta Stone so that we can forever convert from degrees to radians and vice versa.
We know that $\frac{2\pi}{360^0}$ or $\frac{\pi}{180^0}$

From this we can say:
$\frac{x \text{ degrees}}{1 \text{ radian}} = \frac{180^0}{\pi}$
$x \approx 57.3^0$
Therefore, 1 rad is equal to about 57.3^0.

Nov 9-3:06 PM

Example: Convert 40° and 250° into radians.
40° to radians
$\frac{x \text{ radians}}{40^\circ} = \frac{\pi}{180^0}$
$x = \frac{40\pi}{180}$
$x = \frac{2\pi}{9}$
$x \approx 0.698$ rads

Nov 13-10:07 AM

Example: Convert 40° and 250° into radians.
250° to radians
$\frac{x \text{ radians}}{250^\circ} = \frac{\pi}{180^0}$
$x = \frac{250\pi}{180}$
$x = \frac{25\pi}{18}$
$x \approx 4.36$ rads

Nov 9-3:54 PM

Convert π/3, π/2, 3π/4 and 2.3 rads into degrees.

Nov 13-10:15 AM

Convert π/3, π/2, 3π/4 and 2.3 rads into degrees.

π/3 to degrees

$\frac{x \text{ degrees}}{\pi/3} = \frac{180^0}{\pi}$

$x = \left(\frac{\pi}{3}\right)\left(\frac{180^0}{\pi}\right)$

$x = 60^0$

Nov 9-3:58 PM

Convert π/3, π/2, 3π/4 and 2.3 rads into degrees.

π/2 to degrees

$\frac{x \text{ degrees}}{\pi/2} = \frac{180^0}{\pi}$

$x = \left(\frac{\pi}{2}\right)\left(\frac{180^0}{\pi}\right)$

$x = 90^0$

Nov 9-4:03 PM

Convert π/3, π/2, 3π/4 and 2.3 rads into degrees.

3π/4 to degrees

$\frac{x \text{ degrees}}{3\pi/4} = \frac{180^0}{\pi}$

$x = \left(\frac{3\pi}{4}\right)\left(\frac{180^0}{\pi}\right)$

$x = 135^0$

Nov 9-4:06 PM

Convert π/3, π/2, 3π/4 and 2.3 rads into degrees.

2.3 rads to degrees

$\frac{x \text{ degrees}}{2.3} = \frac{180^0}{\pi}$

$x = \frac{(2.3)180^0}{\pi}$

$x \approx 131.8^0$

Nov 9-4:09 PM

Physics timeout! Angular velocity is the rate of change of angular displacement. More specifically it indicates the rotational speed (angular speed) of an object with a directional reference.

Perhaps we should highlight this concept with marching soldiers!

Nov 7-10:00 AM

Example: The London Eye Ferris Wheel has a diameter of 135m and completes one revolution in 30min.
a) Determine the angular velocity ω in radians per second.

b) How far has a rider travelled at 10 minutes into the ride?

Nov 13-10:23 AM

a) Solution
$\omega = \theta/t$

__ s = 30 min * 60s/min
= 1800s

$\omega = \theta/t$
$\omega = 2\pi/1800$
$\omega = \pi/900$ or $\omega \approx 0.00349$ rad/s

Nov 13-10:30 AM

b) Solution
r = 135/2 or r = 67.5m
How many revolutions have occurred?
n = 10 min/30 min
n = 1/3 revolutions.

$d = n(2\pi r)$
$d = \frac{(2\pi(67.5))}{3}$
$d = 45\pi$ m or $d \approx 141.4$m.

Nov 13-10:35 AM