UNIT ONE

Simplifying Expressions and Solving Equations

Designed by Mark Couturier

Department Head at St. Peter Catholic High School

Website: www.habfanforever.com

YouTubeChannel: tangentsinecosine

Twitter: @tansinecosine

Important Term and Terminology

Algebraic Expression: A collection of symbols, including one or more variables and possibly numbers and operation symbols. For example, 3x+6, x, 5x, and 2l – 2w are all algebraic expressions.

Exponent: A special use of a superscript in mathematics. For example, 3^2, the exponent is 2. An exponent is used to denote repeated multiplication. For example, 5^4, means 5x5x5x5.

First-degree equation: An equation in which the variable has the exponent one (1); for example, 5(3x – 1) + 6 = - 20 + 7x + 5.

First-degree polynomial: A polynomial in which the variable has the exponent one (1); for example, 3x – 10.

Integer : Any number within the set {…-4, -3, -2, -1, 0, 1, 2, 3, 4 …}

Inverse Operations: Two operations that "undo" or "reverse" each other. For example, addition and subtraction are inverse operations a + b = c means c – a = b. "Squaring" and "taking the square root" are inverse operations, since, for example, 5^2 = 25 and the (principal) square root of 25 is 5 in other words $\sqrt{25}$ = 5.

Monomial: An algebraic expression with one term. Some examples, $3x^2$, 4x, -7.

Polynomial Expression: An algebraic expression taking the form $a + bx + cx^2 + \ldots$, where a, b, and c are numbers.

Rational Number : Any number than can be expressed as a quotient of two intergers where the divisor is NOT zero for example {…-4, …, $\frac{-3}{2}$, …, $\frac{-1}{12}$, …, 0, 1, … $\frac{54}{23}$, …}

Second-degree polynomial: A polynomial in which the variable in at least one of term has an exponent two (2), and no variable has an exponent greater than two (2). For example, $3x^2 + 10$ or $x^2 - 14x + 48$.

Variable: A symbol used to represent an unspecified number. For example, x and y are variables in the expression x + 2y.

Problem Solving Strategy
The Five-Step Process

LIST:	List all known and unknown variables in your problem.
FORMULA(E):	State any useful formulae that may be of use in your problem.
ALGEBRA:	Is your unknown isolated? If not, use algebra to isolate it.
PLUG-IN	Plug in the known variables into your formula(e).
EVALUATE:	Evaluate the problem and conclude with appropriate units.

Learning Goals in this Unit

By the end of this unit, you will be able to:

- simplify numerical and polynomial expressions in one variable, and solve simple first-degree equations.

- simplify numerical expressions involving integers and rational numbers, with and without the use of technology.

- substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases) [e.g., evaluate $(3/2)^3$ by hand and 9.8^3 by using a calculator])

- describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three [i.e., length, which is one dimensional, can be represented by x; area, which is two dimensional, can be represented by (x)(x) or x^2; volume, which is three dimensional, can be represented by (x)(x)(x), $(x^2)(x)$, or x^3].

- add and subtract polynomials involving the same variable up to degree three [e.g., $(2x + 1) + (x^2 – 3x + 4)$], using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil).

- multiply a polynomial by a monomial involving the same variable to give results up to degree three.

- solve first-degree equations with non-fractional coefficients, using a variety of tools (e.g., computer algebra systems, paper and pencil) and strategies.

- relate your understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify expressions and solve equations.

- substitute into algebraic equations and solve for one variable in the first degree.

Simplifying Expressions

By the end of this unit, you will be able to simplify numerical expressions involving integers and rational numbers, with and without the use of technology.

The Darth Vader versus Luke Skywalker Analogy

Let us define a positive number such as 5 as representing 5 Luke Skywalker's or 5 "good guys". Let us further define a negative number such as -2 as representing 2 Darth Vader. What will happen when the two sets of numbers meet? Well, let's analyze four possibilities.

Scenario # 1: **5 + 2**

In this dull scene, five Luke Skywalker's walk into a room with two other Luke Skywalker's, such that by the end there are 7 Luke Skywalker's. Hence,

$$5 + 2 = 7$$

Scenario # 2: **5 - 2**

In this exciting fight scene, five Luke Skywalker's walk into a room with two Darth Vader's, such that each Luke annihilates a Darth Vader and vice versa. Who wins the fight and by how many? The answer is the "good guys" by 3.

$$5 - 2 = 3$$

Scenario # 3: **- 5 + 2**

Almost identical to the previous fight scene, we now have five Darth Vader's walk into a room with two Luke Skywalker's with the same annihilation condition. Who wins the fight and by how many? The answer is the "bad guys" by 3.

$$-5 + 2 = -3$$

Scenario # 4: **- 5 - 2**

Another dull scene! This time 5 Darth Vader's walk into a room with 2 other Darth Vader's. Will they fight? Of course not! They may be bad guys but they won't fight each other. There are just more bad guys in the room. More specifically; seven (negative or bad guys).

$$-5 - 2 = -7$$

The following exercise is aimed at acquiring your ability to simplify numerical expressions involving integers and rational numbers, with and without the use of technology.

Monetary analogy:

Imagine these four situations and calculate how much money is left in the end.

Scenario # 1: Thomas has 60$. He earns 45$ mowing lawns for his neighbours.

60 + 45
105$

Thomas now has 105$.

Scenario # 2: Stéphanie has 60$. She buys a 45$ blender for her business.

60 - 45
15$

Stéphanie now has 25$.

Scenario # 3: Trevor owes his brother 60$. He earns 45$ cleaning gutters.

- 60 + 45
- 15$

Trevor now has -25$. In other words, he owes that amount to his brother.

Scenario #4: Yasmine owes her brother 60$. She borrows another 45$ to go to a movie.

- 60 - 45
- 105$

Yasmine now has -105$. In other words, she is that amount in debt.

Try these (add $ units at the end to represent dollar values):

1) -5 + 6 = __________ 2) 4 + 2 = __________

3) 5 + 6 – 1 = __________ 4) -4 + 2 = __________

5) -5 - 6 – 1= __________ 6) 4 + 2 – 6 + 5 = __________

Answers (to the Weather Analogy):

1) -4.6 2) 26.9 3) 17.2 4) -12.2 5) -33.51 6) 6.35

Weather analogy:

Imagine these four situations and calculate how warm or cold it is in the end.

Scenario # 1: It is 21.5°C in Miami, Florida. It gets 11.3° C warmer in the afternoon.

21.5 + 11.3
32.8°

It will be 32.8° C in Miami, Florida in the afternoon.

Scenario # 2: It is 21.5°C in Miami, Florida. It gets 11.3° C colder at night.

21.5 – 11.3
10.2°

It will be 10.2° C in Miami, Florida in the afternoon.

Scenario # 3: It is -21.5°C in Orléans, Ontario. It gets 11.3° C warmer thanks to the Sun.

- 21.5 + 11.3
-10.2°

It will now be -10.2° C in Orléans, Ontario.

Scenario #4: It is -21.5°C in Toronto, Ontario. It gets 11.3° C colder thanks to an Arctic cold front.

- 21.5 – 11.3
-32.8°

It will now be -32.8° C in Toronto, Ontario.

Try these (add ° C units at the end to represent Temperature values):

1) -11.1 + 6.5 = __________

2) 14.3 + 12.6 = __________

3) 15.7 + 6.1 – 4.6 = __________

4) -14.1 + 2.0 = __________

5) – 15.4 – 16.2 – 1.91= __________

6) 14.3 + 2.9 –16.2 + 5.35 = __________

Answers (to the Financial Analogy):

1) 1 2) 6 3) 10 4) –2 5) –12 6) 5

In-class task **Name: _________________**

This task is aimed at acquiring your ability to simplify numerical expressions involving integers and rational numbers, with and without the use of technology.

Evaluate the following (involving integers): (4 marks)

1) 7 + 8 = _________ 2) 14 - 5 = _________

3) – 46 + 27 = _________ 4) -17 - 15 = _________

Evaluate the following (involving rational numbers): (4 marks)

1) 8.8 + 2.1 = _________ 2) 21.5 – 7.8 = _________

3) – 14.2 + 2.7 = _________ 4) -91.1 – 65.2 = _________

Josée and the Solar Panel Company (2 marks each for 8 marks)

Josée earned 6,410.57$ installing solar panels on roof tops as a summer job. For her environmental work, she was awarded a 2500$ scholarship to the environmental programme of her choice. Illustrate how much money she has. Don’t forget your units.

To install the solar panels, Josée had to work in her client’s attic where it was 35.2° C. For health and safety reasons, she was obligated to take numerous breaks where she sat on the roof where it was 21.8°C colder. Illustrate what the Temperature was outside on the attic.

Josée’s company needed to borrow 35,000$ to cover their start-up costs. To pay Josée’s salary, the company had to borrow an additional 6,410.57$. Illustrate how much money the company has.

When Josée finished installing the solar panels, the company was paid 27,341.86$. Illustrate how much money the company **now** has.

Task/ Expectation	**Level R**	**Level 1**	**Level 2**	**Level 3**	**Level 4**
Simplify numerical expressions involving integers and rational numbers	Does not simplify numerical expressions involving integers and rational numbers.	Struggles to simplify numerical expressions involving integers and rational numbers.	Sometimes simplifies numerical expressions involving integers and rational numbers.	Usually simplifies numerical expressions involving integers and rational numbers.	Almost always simplifies numerical expressions involving integers and rational numbers.

The Whiner's Club Analogy

Let us define a positive number as a positive person who makes positive comments. Let us further define a negative number as a negative person making negative remarks. What will happen when these sets of people (numbers) will meet? Well, let's analyze four possibilities.

Scenario # 1: **(10)(2)** **or** $\frac{10}{2}$

Person A says, "I got a dog this week, I'm so happy!" Person B says, "That's great! I got parrot this week". In this psychological analogy, two positive people (typically) generate positive attitudes. Hence,

(10)(2)= 20 versus $\frac{10}{2}$ = 5

When two positive numbers are multiplied or divided, they generate a positive outcome!

Scenario # 2: **(10)(-2)** **or** $\frac{10}{(-2)}$

Person A says, "I played fetch with my dog this week, I'm so happy!" Person B says, "My cat ate my parrot this week". In this case, a positive person meets a person who is down or negative, thus resulting is a negative atmosphere. Hence,

(10)(-2)= -20 versus $\frac{10}{(-2)}$ = - 5

When a positive and a negative number are multiplied or divided, they generate a negative outcome!

Scenario # 3: **(-10)(2)** **or** $\frac{(-10)}{2}$

Person A says, "I got a fine for not have a licence on my dog." Person B says, "Oh … I was going to tell you that I got an A in math today". In this case, a negative person meets a happy person, thus again killing the mood and resulting is a negative atmosphere. Hence,

(-10)(2)= -20 versus $\frac{(-10)}{2}$ = - 5

Refer to conclusion in Scenario # 3 … order does not matter!

Scenario # 4: **(-10)(- 2)** **or** $\frac{(-10)}{(-2)}$

Person A says, "Toronto lost again last night!" Person B says, "I saw that! They always lose!" In this case, two negative people are venting, thus comforting themselves (creating a positive feeling). Hence,

(-10)(-2)= 20 versus $\frac{(-10)}{(-2)}$ = 5

When two negative numbers are multiplied or divided, they generate a positive outcome!

Try these (multiplication):

1) (5)(1.13) = __________ 2) (5)(-3) = __________

3) (-7)(4) = __________ 4) (-11)(-14) = __________

Did you know moment!!!

Did you know that when multiplying by 11 where the number being multiplied is less that 10 you need only enter the number twice?

Example:

(-7)(11) = (-7)(10) + (-7)(1)
= -70 - 7
= -77

Try it:

(-8)(11) =

Did you know that when multiplying by 11 where the number being multiplied is between 10 and 99 inclusive that you need only split the number being multiplied and insert the sum of the two numbers?

Example:

(-17)(11) = -(1_7) where the blank is 1+7
= -187
= -77

Try it:

(-27)(11) =

What about (-88)(-11)? Does the trick break down?

(-88)(-11) = (8_8) where the blank is 8+8
= (8_8) which is 16 or (10 + 6)
= 968

(-39)(-11) =

Try these (division)… Answer to TWO decimal places:

1) $\frac{55}{31}$ = __________ 2) $\frac{-52}{11}$ = __________

3) $\frac{41}{-12}$ = __________ 4) $\frac{-5}{-2}$ = __________

Answers (in reverse order):

4) 154 3) -28 2) -15 1) 5.65

4) 2.5 3) -3.42 2) -4.73 1) 1.77

In-class task **Name: __________________**

This task is aimed at acquiring your ability to simplify numerical expressions involving integers and rational numbers, with and without the use of technology.

Evaluate the following (involving integers): (8 marks)

1) (3)(5) = __________

2) (12)(-3) = __________

3) (-17)(11) = __________

4) (-2)(-15) = __________

5) $\frac{55}{11}$ = __________

6) $\frac{-144}{6}$ = __________

7) $\frac{39}{-13}$ = __________

8) $\frac{-15}{-3}$ = __________

Evaluate the following (involving rational numbers): (8 marks)

1) (6.7)(51) = __________

2) (82.6)(-9.1) = __________

3) (-0.42)(6.1) = __________

4) (-22.5)(-1.6) = __________

5) $\frac{198}{11}$ = __________

6) $\frac{-72}{10}$ = __________

7) $\frac{41}{-81}$ = __________

8) $\frac{-45}{-6}$ = __________

Gaston gets Booked (2 marks each for 4 marks)

Gaston bought a series of books to impress Belle, costing him 83.23$. The clerk charges him the 13% HST (Harmonized Sales Tax). Illustrate how much money the clerk will received.

Gaston realizes that he had 12 books at the library which he did not return. He goes to the library with the overdue books and is told to pay a 5.55$ fine for each book. Illustrate how much money Gaston **OWES**? Hint: a fine represents a loss ergo a negative monetary value.

Task/ Expectation	**Level R**	**Level 1**	**Level 2**	**Level 3**	**Level 4**
Simplify numerical expressions involving integers and rational numbers	Does not simplify numerical expressions involving integers and rational numbers.	Struggles to simplify numerical expressions involving integers and rational numbers.	Sometimes simplifies numerical expressions involving integers and rational numbers.	Usually simplifies numerical expressions involving integers and rational numbers.	Almost always simplifies numerical expressions involving integers and rational numbers.

By the end of this class you will be able to substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases) [e.g., evaluate $(3/2)^3$ by hand and 9.8^3 by using a calculator])

The simplified concept of exponents is that it replaces multiplication. In elementary school, you learned that 4+4+4+4+4 was lengthy computationally. You were then taught fast adding or multiplication and replaced the question with 4x5 or (4)(5) which is 20.

Now instead of constantly multiplying numbers like (2)(2)(2)(2)(2), we will utilize fast multiplication, or exponents. We write the above expression to read 2^5 or 2 to the power 5 which equals 32.

Let's try a few.

<u>Squares</u>

$8^2 =$ __________ $(-5)^2 =$ __________ $(\frac{7}{4})^2 =$ __________

<u>Cubes</u>

$5^3 =$ __________ $(-3)^3 =$ __________ $(\frac{7}{4})^3 =$ __________

<u>Other powers</u>

$2^6 =$ __________ $(-4)^{10} =$ __________ $(\frac{7}{4})^4 =$ __________

<u>Know your calculator:</u>

There are a few ways that you can use your calculator. You may have to compute 5^3. If you have a [x^3] button, compute 5[x^3]. You may also have a similar button that looks like [x^y] or [y^x]. Likewise for the square problems, use the [x^2] button.

<u>Answers Squares (in reverse order)</u>

$\frac{49}{16}$, 25, 64

<u>Answers Cubes (in reverse order)</u>

$\frac{343}{64}$, -27, 125

<u>Answer Other powers (in reverse order)</u>

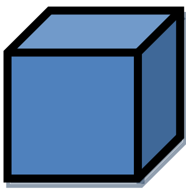
$\frac{2401}{256}$, 1 048 576, 64

Sample problem: A movie theatre wants to compare the volumes of popcorn in two containers, a cube with edge length 8.1 cm and a cylinder with radius 4.5 cm and height 8.0 cm. Which container holds more popcorn?

Did you know? Formula is a Latin word meaning: form or rule. Since it is a Latin word it must be pluralized in Latin, hence the word: **formulae** (pronounced form – u – lie).

Five–Step Problem Solving Strategy in effect.

List (includes drawing the problem and labelling the **known** and **unknown** data)

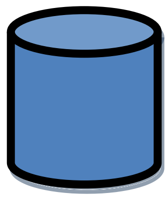


h = 8.1 cm

w = 8.1cm

l = 8.1 cm

r = 4.5cm



h = 8.0 cm

Formulae (find the necessary formulae that you need)

$V = lwh$ $\qquad$ $V = \pi r^2 h$

Algebra (in this case no algebra techniques need to be used since V is already isolated, but since the length, the width and the height are the same, we can say $V = l^3$ or:)

$V = l^3$

Plug-In (Plug in the data from the **List** into the appropriate **Formulae**)

$V = (8.1)^3$ $\qquad$ $V = \pi(4.5)^2(8.0)$

Evaluate and Conclude with Units

$V = (8.1)^3$
$V \approx 531.4\ cm^3$

$V = \pi(4.5)^2(8.0)$
$V = \pi((20.25)(8)$
$V \approx 508.9\ cm^3$

Hence the cube in this case can hold a greater volume than the cylinder.

In-class task **Name: ________________**

This task is aimed to help you acquire the ability to substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases).

Squares

12^2 = __________ $(-6)^2$ = __________ $(\frac{3}{5})^2$ = __________

Cubes

9^3 = __________ $(-6)^3$ = __________ $(\frac{3}{5})^3$ = __________

Other powers

3^6 = __________ $(-8)^{10}$ = __________ $(\frac{3}{5})^4$ = __________

Einstein's Theory of Relativity [Mass-Energy Equivalence]

Find the Energy E [in Joules] given m = 0.000 000 5 kg and c = 299 792 458 m/s [or meters per second], given Einstein's Mass-Energy Equivalence Formula.

$$\mathbf{E = mc^2}$$

Quadratic Warm-up

Given the value of t = 3, find the value of h given the quadratic.

$$\mathbf{h = -4.9t^2 + 10t + 3}$$

Task/ Expectation	**Level R**	**Level 1**	**Level 2**	**Level 3**	**Level 4**
Demonstrate the ability to substitute into and evaluate algebraic expressions involving exponents.	Does not demonstrate the ability to substitute into and evaluate algebraic expressions involving exponents.	Struggles to demonstrate the ability to substitute into and evaluate algebraic expressions involving exponents.	Sometimes demonstrates the ability to substitute into and evaluate algebraic expressions involving exponents.	Usually demonstrates the ability to substitute into and evaluate algebraic expressions involving exponents.	Almost always demonstrates the ability to substitute into and evaluate algebraic expressions involving exponents.

By the end of this class you will be able to describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three [i.e., length, which is one dimensional, can be represented by x; area, which is two dimensional, can be represented by (x)(x) or x^2; volume, which is three dimensional, can be represented by (x)(x)(x), $(x^2)(x)$, or x^3.

<u>Algeo-Tiles</u>

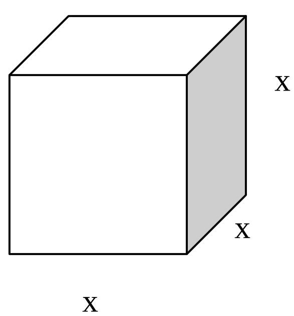
The following shapes are often referred to as Algeo-tiles. We can use these to solidify your understanding that a number can be represented by a square such that its area is the value 1. Check: Area = (length)(width) or A = (1)(1)

An x, or an unknown value x, can be represented by a rectangle such that its area is the value x. Check : Area = lw or A = (1)(x) or A = x

An x^2, or an unknown value x squared, can be represented by a square such that its area is the value x^2. Check: Area = lw or A = (x)(x) or A = x^2.

Note that the black shapes represent positive values whereas the red shapes represent negative values. If one of each colour is present, they will cancel themselves out.

Extension: What do you think x^3 would look like?



Dimension Theory

Dimension	**Diagram**	**Concept**	**Units**
0 – D	•	point	unitless

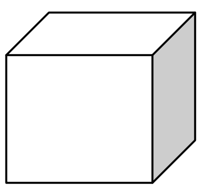
0-D has no lengths to measure, hence it is unitless. Motion in a point world would be non-existent.

1 – D	________	length	cm, m, inches, ft

1-D can be represented by a line, hence it can have a measurable length. Think of Perimeter which is the length around a shape. Also, look at the exponent in the units and recall that when the exponent is 1, we do not integrate that within the units.

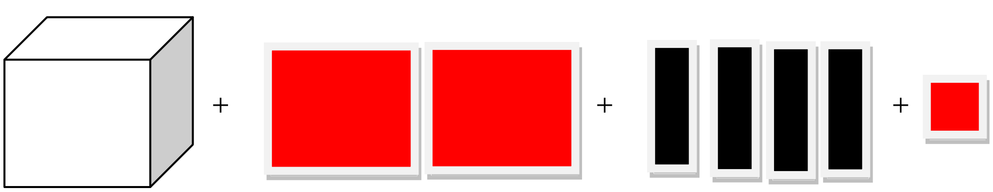
2 – D		area	cm^2, m^2, sq in, sq ft

2-D is a surface, hence has a measurable length and width. Think of Area as being the calculation of two lengths hence (4m)(5m) where the "m times m" makes the second dimension hence m^2. If you say m^2, you know it is 2-D (m squared).

3 – D		volume	cm^3, m^3, cu in, cu ft

3-D is a volume (cube), hence has a measurable length, width and height. Think of Volume as being the calculation of three lengths hence (4m)(5m)(2m) where the "m times m times m" makes the third dimension hence m^3. If you say m^3, you know it is 3-D (m cubed).

Example: Illustrate the following expression using algeo-tiles: $x^3 - 2x^2 + 4x - 1$



<u>In-class task</u> **Name: ________________**

This task is aimed at acquiring the ability to describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three [i.e., length, which is one dimensional, can be represented by x; area, which is two dimensional, can be represented by (x)(x) or x^2; volume, which is three dimensional, can be represented by (x)(x)(x), (x^2)(x), or x^3.

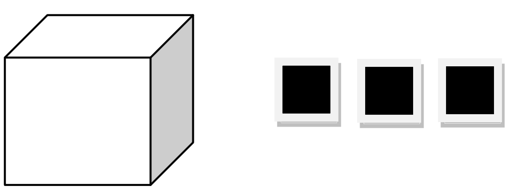
<u>Use algeo-tiles to represent the following algebraic expressions</u>

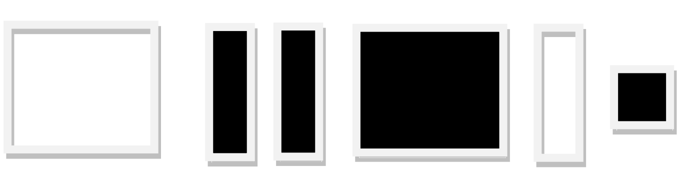
$3x – 5 + 2x – 1$

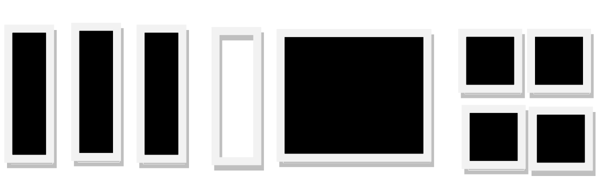
$x^2 – 3x + 2 + x^2 – 2x – 2$

$x^3 – 2x + 3$

<u>Write the algebraic expression Use algeo-tiles to present the following algebraic statements</u>







Task/ Expectation	Level R	Level 1	Level 2	Level 3	Level 4
Describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three.	Does not describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three.	Struggles to describe the relationship between the algebraic and geometric representations of a single-variable term up to degree three.	Sometimes describes the relationship between the algebraic and geometric representations of a single-variable term up to degree three.	Usually describes the relationship between the algebraic and geometric representations of a single-variable term up to degree three.	Almost always describes the relationship between the algebraic and geometric representations of a single-variable term up to degree three.

By the end of this class you will be able to add and subtract polynomials involving the same variable up to degree three [e.g., $(2x + 1) + (x^2 – 3x + 4)$], using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil).

A Day at the Farm

To understand the concept of like and unlike terms we need to conceptually understand what can be combined and what cannot. In this analogy, let x represent a horse and let y represent a duck. In this example, pretend you are looking at a field with the following:

Literally, we can say:

"There is a duck plus two horses and two more ducks".

Again literally, we can further simply our statement to say:

"There are 3 (three) ducks plus 2 (two) horses".

Now let us do this algebraically:

 **+2**  **+ 2** 

Note that we do not place a "1" in front of the lone duck as it is understood that there is one there. Can we simplify this further?

3  **+2** 

Compare this last statement with the "literal" English statement.

Recall now that we said that x represented horses and y represented ducks. Our previous algebraic statement now becomes:

3y + 2x

Your turn to try:

Simplify the following:

Write what you see “literally”:

Write a “let” statement by assigning a letter to represent the dolphin? the manatee?

Write what you see algebraically:

Simplify your algebraic statement.

In the previous example, you noticed that we cannot combine ducks and horses nor dolphins and manatees. You should also notice that we cannot combine x's and y's.

Example: $6x - 4y + 9x - y$

Note that we have two sets of x's and two sets of y's. Let us combine them together.

$$6x + 9x - 4y - y$$

$$15x - 5y$$

We must stop here as these are now UNLIKE terms.

Harder Example: $3x - 4y + 9 - 7x - 8y + z - 7$

In this case, we have two sets of x's, two sets of y's, two numbers and a lone z. We can only combine (add) the x's with the x's, the y's with the y's, the z's with the z's and the lone numbers with lone numbers

Hence we can combine the following as follows:

$$3x - 7x - 4y - 8y + z + 9 - 7$$

Note that we take care to retain the term's sign (+ or -). Now we simplify.

$$-4x - 12y + z + 2$$

We must stop here. These are now ALL unlike terms and cannot be combined.

Challenging Example: $3.1x - 4.8y + 9.8 - 7.5x - 8.3y + 1.4z - 7.9$

The only difference between this example and the previous one is the use of rational numbers as opposed to integers.

$$3.1x - 7.5x - 4.8y - 8.3y + 1.4z - 7.9 + 9.8$$

$$-4.4x - 13.1y + 1.4z + 1.9$$

Expectation example: $(2x + 1) + (x^2 - 3x + 4)$

Note that it is customary to write the term with the highest degree first (the term with the largest exponent value).

$$x^2 + 2x - 3x + 1 + 4$$

$$x^2 - x + 5$$

This task is aimed at acquiring the ability to add and subtract polynomials involving the same variable up to degree three [e.g., $(2x + 1) + (x^2 – 3x + 4)$], using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil).

Try these (use any method):

1) $5x + 6x – x$

2) $4x + 2y – 6y + 5x$

3) $5x^2 + 6x – x^2$

4) $4x^2 + 2y^2 – 6y + 5x^2$

5) $5 + 6x – x$

6) $4 + 2y^2 – 6y^2 + 5$

Answers:

1) $10x$ 2) $9x – 4y$ 3) $4x^2 + 6x$ 4) $9x^2 + 2y^2 – 6y$

5) $5 + 5x$ 6) $9 – 4y^2$

<u>In-class task</u> **Name: ________________**

This task is aimed at acquiring the ability to add and subtract polynomials involving the same variable up to degree three [e.g., $(2x + 1) + (x^2 – 3x + 4)$], using a variety of tools (e.g., algebra tiles, computer algebra systems, paper and pencil).

<u>Try these (use of algeo-tiles option available):</u>

1) $2x + 6x – 9x$

2) $4x + 2 – 5x + 5$

3) $15x^2 + 4x – 12x^2$

4) $x^2 + 3y^2 – y + 7x^2$

5) $5x + 16x – 7x$

6) $4y + 2y – 6y^2 + 5$

<u>Simply the following :</u>

Jean-François is at the Drive-Thru and his friends are disorganized in their order of Hot Dog Meals and Hamburger Meals. They order the following:

You think that this order is ridiculous and that there is a simpler way to make this order. Using “Let Statements” and algebraic language, how would you place this order to be more time efficient?

Task/ Expectation	Level R	Level 1	Level 2	Level 3	Level 4
Adds and subtracts polynomials involving the same variable up to degree three.	Does not add or subtract polynomials involving the same variable up to degree three.	Struggles to add and subtract polynomials involving the same variable up to degree three.	Sometimes adds and subtracts polynomials involving the same variable up to degree three.	Usually adds and subtracts polynomials involving the same variable up to degree three.	Almost always adds and subtracts polynomials involving the same variable up to degree three.

By the end of this class you will be able to multiply a polynomial by a monomial involving the same variable to give results up to degree three.

It is important to recognize that the rules of addition and subtraction are different from the rules of multiplication. Hence, $x+x = 2x$ but $(x)(x) = x^2$.

Note that : $(x)(x) = x^2$.

Another way to think … $(x^1)(x^1) = x^{1+1}$
or … $(x^1)(x^1) = x^2$

Why? Let's reason this out before stating the first **Exponent Law**. Let's say that $x = 5$.

$(x)(x) = x^2$
$(5)(5) = 5^2$ or 25.

<u>Exponent Law of Multiplication</u>

$$\mathbf{(x^a)(x^b) = x^{a+b}}$$

What about this one?

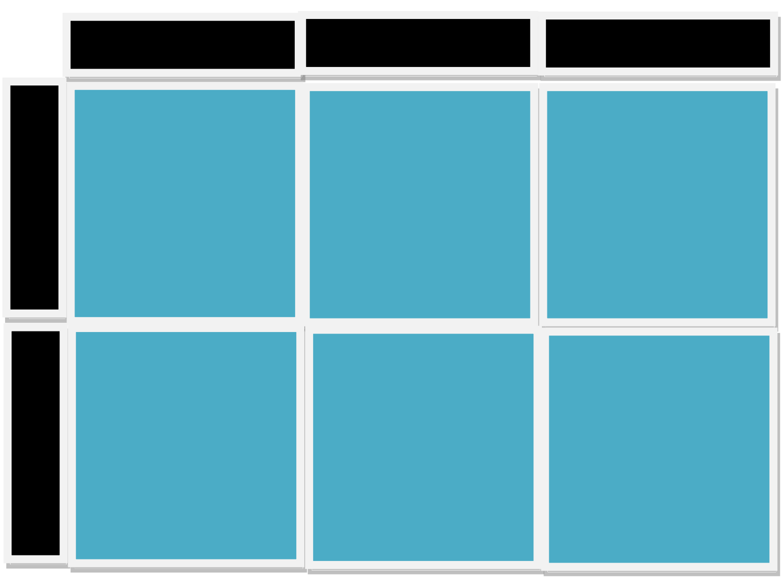
$(2x)(3x) = 6x^2$

Another way to think … $(x^1)(x^1) = x^{1+1}$
or … $(x^1)(x^1) = x^2$

In this case, we are still dealing with $(x)(x)$, which explains the x^2. How do we get the 6? Recall that we are multiplying so we apply the following general rule:

Multiply the coefficients and add the exponents associated with the terms.

Why does it work?



(the row represents 3x's)

How many shapes does it take to fill in the rectangle?

In this case, $6x^2$'s are needed to fill the rectangle.

Blue is used for effect.

(the Column represents 2x's)

This task is aimed at helping you acquire the ability to multiply a polynomial by a monomial involving the same variable to give results up to degree three.

Recall:

Exponent Law of Multiplication

$$(x^a)(x^b) = x^{a+b}$$

Try these (use any method):

1) $(4x)(5x)$ = _________

2) $7(11x)$ = _________

3) $(4x)(5x) + x$ = _________

4) $7(11x) + 1$ = _________

Build-up examples: Cover the right side of the page which is reversed.

$(2x)(5)$ = _________	$(2x)(5x^2) = 10x^3$
$(2x)(5x)$ = _________	$(2x)(5x) = 10x^2$
$(2x)(5x^2)$ = _________	$(2x)(5) = 10x$

Why?

Let's review the previous notion that:

$$(x)(x) = x^{1+1}$$
$$(x^1)(x^1) = x^2$$

The first case is simply

$$(2x)(5) = (2x^1)(5) \text{ or 5 2x's.}$$
Hence, 10x or 10x's.

The second case is somewhat as before:

$$(2x)(5x) = (2x^1)(5x^1) \text{ or } 10x^{1+1}.$$
$$\text{Hence, } (2x^1)(5x^1) = 10x^2$$

The last case will therefore become:

$$(2x)(5x^2) = (2x^1)(5x^2) \text{ or } 10x^{1+2}.$$
$$\text{Hence, } (2x^1)(5x^2) = 10x^3$$

What do you think would happen with these?

$(2x^2)(5x^2)$ = ________ $(2x^3)(5x^2)$ = ________ $(2x^{21})(5x^{32})$ = ________

Answers in reverse: $10x^{53}$, $10x^5$, $10x^4$

The Distributive Law

By the end of this class you will be able to multiply a polynomial by a monomial involving the same variable to give results up to degree three.

In this task, we will look at at variety of methods to analyze: 3(2x+1)

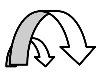
Elementary Algebra

3(2x+1) equates to 3 of what is in the bracket. Hence,

3(2x+1) = (2x+1) (2x+1) (2x+1)
= 2x + 2x + 2x + 1 + 1 + 1
= 6x + 3

The Distributive Law

a(bx + c) = abx + ac



3(2x+1) = 3(2x+1)

= 6x + 3

Note that the 3 outside the brackets gets multiplied by all the terms in the brackets.
Hence 3 times 2x is 6x and 3 times 1 is 3.

Try these: (You are encouraged to use arrows to apply the Distributive Law):

6(x + 1) = ______________________

4x(x-3) = ______________________

6x(x + 1) = ______________________

$4x^2$(x-3) = ______________________

$6x^2$(x + 1) = ______________________

4(x^2-3x) = ______________________

2($5x^2$ – 4x) – 2(3x-7) = __________________

2x($5x^2$ – 4x – 2) = __________________

Answers:

6(x + 1) = 6x+6
6x(x + 1) = $6x^2$ + 6x
$6x^2$(x + 1) = $6x^3$ + $6x^2$
2($5x^2$ – 4x – 2) = $10x^2$ – 14x + 14

4x(x-3) = $4x^2$-12x
$4x^2$(x-3) = $4x^3$-$12x^2$
4(x^2-3x) = $4x^2$-12x
2x($5x^2$ – 4x – 2) = $10x^3$ – $8x^2$ – 4x

Using Algeo-tiles

3(2x+1) =

Setup: Use a column to represent the 3 and a row to represent the 2x+1 using Algeo-tiles.

3(2x+1) =

Now imagine that the 3 and the (2x+1) is the contour of a rectangle which you are now to fill in using algeo-tiles

Blue is used for effect only. Count the blue x's and the blue one's Note 6x + 3.

Try this one: (using Algeo-tiles)

4(3x+2)

Self commentary: Which of the three methods (elementary, Algeo-tiles and/or Distributive Law) did you find the most effective?

In-class task **Name: _________________**

This task is aimed at acquiring the ability to multiply a polynomial by a monomial involving the same variable to give results up to degree three.

Simplify the following

1) $11(2x) =$ _________

2) $-7(4x) =$ _________

3) $(6x)(3x) =$ _________

4) $3(-11x) =$ _________

5) $(3x^2)(7x) =$ _________

6) $(-4x)(12x^2) =$ _________

7) $(x^2)(-9x) =$ _________

8) $(-3x^2)(-10x) =$ _________

Expand the following (some blanks removed to encourage multiple steps)

9) $2(7x-1) =$ _________

10) $4x(3x - 5) =$ _________

11) $x^2(x - 4) =$ _________

12) $6(3x^2 -10x - 1) =$ _______________

13) $4(x-5) + 5(x-8) =$

14) $6(3x^2 -10x - 1) =$ ________________

15) $7(2x–4) – 5(x–3) =$

16) $–2x(7x^2 – 5) + 4(3x^3 – 5x) =$

Use Algeo-tiles to illustrate the solution below

$3(2x+4)$

Task/ Expectation	Level R	Level 1	Level 2	Level 3	Level 4
Multiply a polynomial by a monomial involving the same variable to give results up to degree three.	Does not multiply a polynomial by a monomial involving the same variable to give results up to degree three.	Struggles to multiply a polynomial by a monomial involving the same variable to give results up to degree three.	Sometimes multiplies a polynomial by a monomial involving the same variable to give results up to degree three.	Usually multiplies a polynomial by a monomial involving the same variable to give results up to degree three.	Almost always multiplies a polynomial by a monomial involving the same variable to give results up to degree three.

By the end of this class you will be able to solve first-degree equations with non-fractional coefficients, using a variety of tools (e.g., computer algebra systems, paper and pencil) and strategies.

Algebra

Up to now, you have been doing algebra in a subliminal manner in the form:

$4 + 5 = x$ $15 - 9 = y$ $(4)(7) = z$ $\frac{30}{6} = a$

In these cases, the problem is simply an evaluation issue rather than an algebraic one. We will now utilize algebra or reverse operations to obtain the value we desire for questions of the nature:

$x + 15 = 19$ $y - 5 = 10$ $10z = 40$ $\frac{a}{2} = 16$

One Step-Algebra (involving addition)

Solve for x. To do this we need to **ISOLATE** the variable x.

$$x + 15 = 19$$

Our problem here is that we have a positive 15 on the left side. To get rid of it, we must subtract 15 from the left and therefore from the right side of the equal side.

$$x + 15 - 15 = 19 - 15$$

What you do to one side of the equal sign, you must do to the other side.

This cancels the 15's (since 15-15 = 0). Therefore,

$$x = 4$$

We can now check our work to verify that this is true with a "left side right side" check. Create the cross below as labelled and re-write the problem as initially stated without an equal sign. In the next line, enter the value of x which you believe to be true.

Left Side	Right Side
x + 15	19
(4) + 15	
19	

Since the left side equals the right side, we know that x = 4 is the correct answer.

<u>One Step-Algebra (involving subtraction)</u>

Solve for y. To do this we need to **<u>ISOLATE</u>** the variable y.

$$y - 5 = 10$$

Now the problem here is that we have a negative 5 on the left side. To get rid of it, we must add 5 to the left and therefore to the right side of the equal side.

$$y - 5 + 5 = 10 + 5$$

What you do to one side of the equal sign, you must do to the other side.

This cancels the 5's (since -5 + 5 = 0). Therefore,

$$x = 15$$

We again wish to confirm our solution with a "left side right side" check.

Left Side	Right Side
y – 5	10
(15) – 5	
10	

Since the left side equals the right side, we know that y = 15 is the correct answer.

<u>One Step-Algebra (involving multiplication)</u>

Solve for z. **<u>ISOLATE</u>** the variable z.

$$10z = 40$$

In this case, we need to divide **both** sides by 10 because $\frac{10}{10} = 1$ which will yield 1z or just z.

$$\frac{10}{10}z = \frac{40}{10}$$

Hence,

$$z = 4$$

Check:

Left Side	Right Side
10z	40
10(4)	
40	

The left side equals the right side, so we are correct; z = 4.

One Step-Algebra (involving division)

Solve for a. **ISOLATE** the variable a.

$$\frac{a}{2} = 16$$

In this case, we need to multiply **both** sides by 2 because $\frac{2}{2}$ = 1 which will yield 1a or just a.

$$\frac{2}{2}a = (2)(16)$$

Hence,

$$a = 32$$

Check:

Left Side	Right Side
$\frac{a}{2}$	16
$\frac{32}{2}$	
16	

The left side equals the right side, so we are correct; z = 32.

CAUTION

Something to watch out for … In both these cases students often make errors because they don't recognize for which numbers are negative.

15 – t = 17 –11t = 132

In both cases, we still solve for t by **ISOLATING** the unknown.

On the left we subtract 15 from both sides. On the right, we divide -11 from both sides.

$15 - t - 15 = 17 - 15$ $\frac{-11t}{-11} = \frac{132}{-11}$

We need to be careful when simplifying our work. Note that on the left, the t is still negative. Numerically, it is like saying (–1)t . We can divide both sides by negative –1 to get rid of it.

– t = 2 t = – 12

$$\frac{-t}{-1} = \frac{2}{-1}$$

Hence, t = – 2

This is task is aimed at helping you acquire the ability to solve first-degree equations with non-fractional coefficients, using a variety of tools (e.g., computer algebra systems, paper and pencil) and strategies.

Your turn: Solve for the unknown variable. (Be sure not to use Trial & Error)

5x = 35

$\frac{y}{7} = 8$

6 + t = 45

p – 12 = 16

7 – g = 4

– 7g = 56

h + 3.12 = 13.56

x – 54 = 161

Answers: x = 7, y = 56, t = 39, p = 28, g = 3, g = – 8, h = 10.44, x = 215.

<u>In-class task</u> **Name: _________________**

This is task is aimed at helping you acquire the ability to solve first-degree equations with non-fractional coefficients, using a variety of tools (e.g., computer algebra systems, paper and pencil) and strategies.

Solve for x in each of the following. Don't forget **SAMDEB**. Be sure to check your answers.

x – 7 = 11

5x = 45

$\frac{x}{9}$ = 3

x + 12 = – 13

– 8x = 32

12 – x = 31

Task/ Expectation	Level R	Level 1	Level 2	Level 3	Level 4
Solve first-degree equations with non-fractional coefficients.	Does not solve first-degree equations with non-fractional coefficients.	Struggles to solve first-degree equations with non-fractional coefficients.	Sometimes solves first-degree equations with non-fractional coefficients.	Usually solves first-degree equations with non-fractional coefficients.	Almost always solves first-degree equations with non-fractional coefficients.

Two Step-Algebra

Imagine a situation such as the following:

$3(4) - 5 = y$

To determine the value of y, you apply BEDMAS order of operation. You would do this because y is ISOLATED and the operations are all on one side. Following BEDMAS would yield y = 7.

However, if the situation were:

$3x - 5 = 7$ where we need to solve for x. We need to **ISOLATE** the variable x.

$$3x - 5 = 7$$

Our problem here is that we have a negative 5 on the left side and a multiple of 3. Since we are working in reverse, we need to use **SAMDEB**. This is just BEDMAS backwards. Hence, we must get rid of the **SUBTRACTION** first. To get rid of it, we must add 5 to the left and therefore to the right side of the equal side.

$$3x - 5 + 5 = 7 + 5$$

What you do to one side of the equal sign, you must do to the other side.

This cancels the 5's (since – 5 + 5 = 0). Therefore,

$$3x = 12$$

Now we have a simple one step algebra problem here.

$$\frac{3x}{3} = \frac{12}{3}$$

Hence;

$$x = 4$$

We can now check our work to verify that this is true with a "left side right side" check.

Left Side	Right Side
3x – 5	7
3(4) – 5	
12 – 5	
7	

Since the left side equals the right side, we know that x = 4 is the correct answer.

Sample problem: Solve $2x + 7 = 6x – 1$

In this case, it would be wise to move all the x's to the left side and all the numbers to the right side. To do this, we must subtract both sides by 7 and both sides by 6x.

$$2x + 7 = 6x - 1$$

$$2x + 7 - 7 - 6x = 6x - 1 - 7 - 6x$$

The consequence of this is that is cancels (or eliminates) the 7's on the left and the x's on the right (since 7 – 7 = 0 as does 6x – 6x = 0)

$$2x - 6x = -1 - 7$$

Let us simplify this by combining like terms.

$$-4x = -8$$

We now have a single step algebra problem now where we must get rid of a multiple of -4 .

$$\frac{-4x}{-4} = \frac{-8}{-4}$$

Note that $\frac{-4}{-4}$ = 1 (recall double negatives)

$$x = 2$$

Check:

Left Side	Right Side
2x + 7	6x – 1
2(2) + 7	6(2) – 1
4 + 7	12 – 1
11	11

Since the left side equals the right side, we know that x = 2 is the correct answer.

This is task is aimed at helping you acquire the ability to solve first-degree equations with non-fractional coefficients, using a variety of tools and strategies.

Solve for x in each of the following. Don't forget **SAMDEB**. Be sure to check your answers.

$5x - 1 = 19$	$7x - 1 = 3x + 15$
$\frac{x}{3} - 4 = 11$	$11x + 7 = 5x - 17$
$10x - 7x + 3 = -7 + 2x$	$16x - 9 = 23$

Answers: $x = 4$, $x = 4$, $x = 45$, $x = -4$, $x = -10$, $x = 2$

<u>In-class task</u> **Name: _________________**

This is task is aimed at helping you acquire the ability to solve first-degree equations with non-fractional coefficients, using a variety of tools and strategies.

Solve for x in each of the following. Don't forget **SAMDEB**. Be sure to check your answers.

$2x - 6 = 16$

$5x - 9 = 2x + 21$

$\frac{x}{9} - 3 = 5$

$8x + 12 = 2x - 12$

$-8x + 6 = -4 + 2x$

$9x - 3 = 42$

Task/ Expectation	Level R	Level 1	Level 2	Level 3	Level 4
Solve first-degree equations with non-fractional coefficients.	Does not solve first-degree equations with non-fractional coefficients.	Struggles to solve first-degree equations with non-fractional coefficients.	Sometimes solves first-degree equations with non-fractional coefficients.	Usually solves first-degree equations with non-fractional coefficients.	Almost always solves first-degree equations with non-fractional coefficients.

By the end of this class you will be able to relate your understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify expressions and solve equations.

<u>Squaring vs Square Rooting</u>

The function of squaring is a simplified way to say that a number, say x, is being multiplied by itself, hence (x)(x) or x^2. When we say x^2, we say "x squared". Recall the Algeo-tile representation of x^2.

x  where the area of this "square" is A = lw or A = (x)(x) or $A = x^2$.

x

Now let's take this example to the next level. Let's say that we know that the area of the square is A = 169 cm^2 and we wish to find the length and width (which are equal).

To do this, we need the "square root" of the area. The square root is simply the inverse operation of the square. Just like adding cancels subtraction and multiplying cancels division; "square rooting" cancels squaring.

In this example, we have, A = 169 cm^2, and l = w, hence,

$$lw = A$$
$$l^2 = 169$$

We will now square root BOTH sides of the equation. Recall the adage:

What you do to one side of the equal sign, you must do to the other side.

$$\sqrt{l^2} = \sqrt{169}$$
$$l = 13 \text{ cm}$$

Can you find your "square root" button on your calculator? In most cases, it looks like: $\sqrt{x}$.

Calculate the area of a square with length l = 17 cm. Show what you did with your calculator.

Find the length of a square with Area A = 215 cm^2. Show what you did with your calculator.

<u>**Group work Activity**</u>

This task is aimed at helping you acquire the ability to relate the understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify expressions and solve equations.

<u>**Physics – Radiation and Half Life**</u>

In physics, radioactive substances lose half their radioactive mass every “half-life”. It follows the half-life formula:

$$\mathbf{M_f = M_i\left(\frac{1}{2}\right)^n}$$

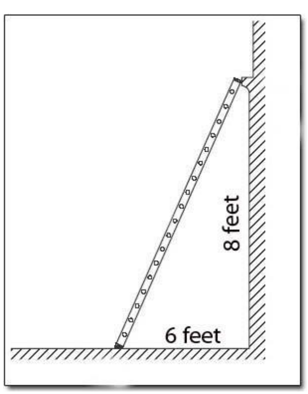
where M_f is the final mass, M_i is the initial mass and n is the number of “half-lives”.

If a 650g sample of Californium-253, which has a half life of 17.81 days, is left on a desk for 2 half-lives or 35.62 days, how much of that substance will be left?

<u>**Pythagorean Theorem**</u>

Recall Pythagorean Theorem which states that : $\mathbf{c^2 = a^2 + b^2}$

Now let’s consider the unknown length of a ladder leaning against a wall. The top of the ladder is 8 feet from the ground and the base of the ladder is 6 feet from the wall. How long is the ladder?



<u>In-class task</u> **Name: _________________**

This task is aimed at helping you acquire the ability to relate the understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify expressions and solve equations.

A student has designed a square garden plot as shown below and is sold in two sizes.

The small version has a length of 2.5 ft. What is the area?

A gardener wishes to divide the plot in half using the diagonal. What is the length of the diagonal? Use Pythagorean Theorem ($\mathbf{c^2 = a^2 + b^2}$).

The large version has an area of 72.25 sq ft (square feet). What is the length of the plot?

Task/ Expectation	Level R	Level 1	Level 2	Level 3	Level 4
Relate the understanding of inverse operations to squaring and taking the square root, and apply inverse operations to simplify expressions and solve equations.	Does not relate the understanding of inverse operations to squaring and taking the square root, and applying inverse operations to simplify expressions and solving equations.	Struggles to relate the understanding of inverse operations to squaring and taking the square root, and applying inverse operations to simplify expressions and solving equations.	Sometimes relates the understanding of inverse operations to squaring and taking the square root, and applying inverse operations to simplify expressions and solving equations.	Usually relates the understanding of inverse operations to squaring and taking the square root, and applying inverse operations to simplify expressions and solving equations.	Almost always relates the understanding of inverse operations to squaring and taking the square root, and applying inverse operations to simplify expressions and solving equations.

By the end of this class you will be able to substitute into algebraic equations and solve for one variable in the first degree.

Radius of a Circle

Find the radius of the circle below given that the Area is A ≈ 113.1 mm^2.

Five–Step Problem Solving Strategy in effect.

List: A ≈ 113.1 mm^2; r = unknown

Formula: $A = \pi r^2$

Plug-in:

$A = \pi r^2$

$113.1 = \pi r^2$

Algebra: We need to solve for r. To do so, we must divide both sides by π.

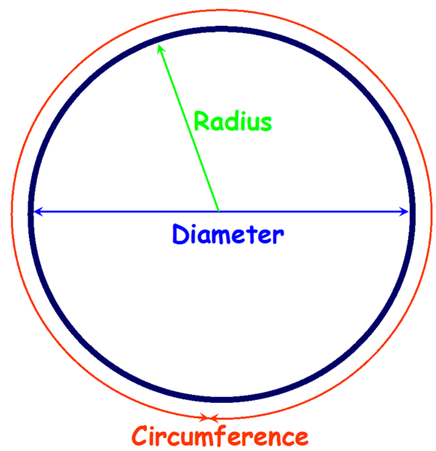
$\frac{113.1}{\pi} = \frac{\pi r2}{\pi}$

$r^2 \approx 36$

$\sqrt{r^2} \approx \sqrt{36}$

Evaluate and Conclude with Units

r = 6 mm



Sample problem: The perimeter of a rectangle can be represented as P = 2l + 2w. If the perimeter of a rectangle is 59 cm and the width is 12 cm, determine the length.

Five–Step Problem Solving Strategy in effect.

List: P = 59 cm; w = 12cm, l = unknown.

Formula: P = 2l + 2w

Algebra: We need to solve for l.

P = 2l + 2w — Is l **ISOLATED**? If not, what is in the way? An "adding" or positive 2w and a multiple of 2.

P – 2w = 2l + 2w – 2w — Follow **SAMDEB**: subtract 2w first.

P – 2w = 2l — Now divide both sides by 2.

$\frac{P - 2w}{2} = \frac{2l}{2}$

l = $\frac{P - 2w}{2}$

Plug-in: Let's plug in the values from our list.

l = $\frac{59 - 2(12)}{2}$ — Remember now to use **BEDMAS**.

<u>**Evaluate and Conclude with Units**</u>

l = $\frac{(59 - 24)}{2}$ — Multiplication in the brackets first.

l = $\frac{35}{2}$ — Subtraction in the brackets second.

l = 17.5 cm — Division at the end.

Alternative Approach (Switching Plug-in and Algebra)

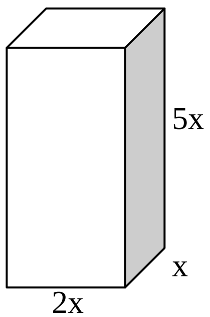
P = 2l + 2w
59 = 2l + 2(12)
59 – 24 = 2l + 24 – 24
35 = 2l

$\frac{35}{2} = \frac{2l}{2}$

l = 17.5 cm

Volume of a Rectangular Prism

Given the rectangular prism below of height 5x, length 2x and width x, create a formula to determine the volume.



List: V = unknown, l = 2x, w = x and h = 5x.

Formula: V = lwh

Plug-in: V = (2x)(x)(5x)

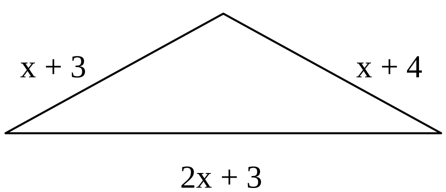
Evaluate: $V = 10x^3$

If x = 2 cm, what is the Volume?

$V = 10x^3$
$V = 10(2)^3$ Remember BEDMAS
$V = 10(8)$
$V = 80\ cm^3$

Perimeter of a Triangle

Given the triangle below, create a formula to determine its perimeter.



P = a + b + c
P = (x + 3) + (x + 4) + (2x + 3)
P = 4x + 10

If the Perimeter is 50m, what is the length of each side?

P = 4x + 10
50 = 4x + 10
50 – 10 = 4x + 10 – 10
40 = 4x
$\frac{40}{4} = \frac{4x}{4}$
x = 10

Hence the sides lengths are (x+3) or (10+3) = 13cm, (x+4) or (10+4) = 4cm and (2x+3) or (2(10)+3) = 23cm.

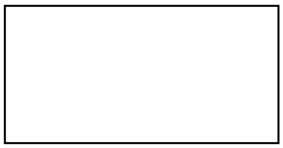
This task is aimed at helping you acquire the ability to substitute into algebraic equations and solve for one variable in the first degree.

<u>Width of a Rectangle</u>

Given the rectangle below whose Perimeter P is 50cm, find the width, w.

15cm

w



$P = 2l + 2w$

<u>Height of a Pyramid</u>

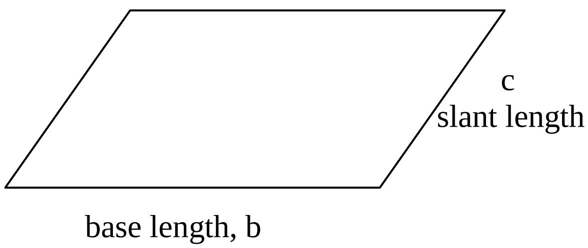
While visiting the pyramids in Egypt, you hear the tour guide state that it took a volume of 6,000,000 cu ft (cubic feet) in stones to make the pyramid. While walking about its square base, you observe its length to be 200ft. How high is the pyramid?

$$V = \frac{l^2 h}{3}$$

This task is aimed at helping you acquire the ability to substitute into algebraic equations and solve for one variable in the first degree.

Base of a Parallelogram

Given the parallelogram below whose slant length c = 2m and whose Perimeter P is 9cm, find the base length, b.   Note that the Perimeter is P = 2b + 2c.



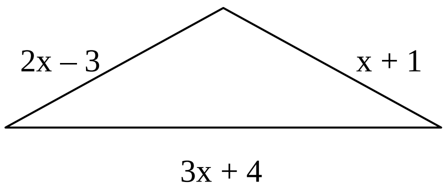
Height of an Ice Cream Cone

Given an ice cream cone that can hold a volume of 65.5 cm^3 of ice cream (inside the cone of course) and whose radius is 2.5 cm, what is its height of the cone?

$$V = \frac{\pi r^2 h}{3}$$

Perimeter of a Triangle

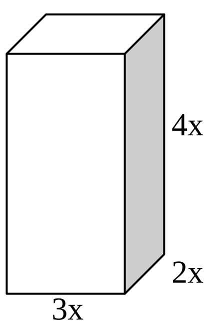
Given the triangle below, **create a formula** to determine its **perimeter**.



Using your formula, find the length of each side if the Perimeter is 32 cm.

Volume of a Rectangular Prism

Given the rectangular prism below of height 4x, length 3x and width 2x, **create a formula** to determine the **volume**.



Using your formula, find the volume given that x = 1.45 cm.

Task/ Expectation	Level R	Level 1	Level 2	Level 3	Level 4
Substitute into algebraic equations and solve for one variable in the first degree.	Does not substitute into algebraic equations and solve for one variable in the first degree.	Struggles to substitute into algebraic equations and solve for one variable in the first degree.	Sometimes substitutes into algebraic equations and solves for one variable in the first degree.	Usually substitutes into algebraic equations and solves for one variable in the first degree.	Almost always substitutes into algebraic equations and solves for one variable in the first degree.

MOCK EQAO

While experimenting with a toy rocket, Daniel determines that he can model the rocket's height, h, in metres, with respect to time, t, in seconds, using the equation:

$$h = \frac{1}{2}t^2$$

Which calculation correctly finds the value of h when t = 10?

A) $h = \frac{1}{2} \times 10^2$
$= 5^2$
$= 25$

B) $h = \frac{1}{2} \times 10^2$
$= \frac{1}{2} \times 20$
$= 10$

C) $h = \frac{1}{2} \times 10^2$
$= \frac{1}{2} \times 100$
$= 50$

D) $h = \frac{1}{2} \times 10^2$
$= \frac{1}{4} \times 100$
$= 25$

A rectangular field has a perimeter of (10a – 6) metres and a width of 2a metres.

Which expression represents the **length** of this field?

A) $8a - 6$ B) $12a - 6$ C) $3a - 3$ D) $3a^2 - 3$

Simplify the following expression:

$$3x(2x + 3) - 5x$$

A) $6x^2 - 5x + 3$ B) $6x^2 - 6x$ C) $15x^2 - 5x$ D) $6x^2 + 4x$

Meghan has been asked to determine the value of the numerical expression below:

$$\frac{2^{400}}{2^{396}} - 2^3$$

Which of the following is the value of Meghan's expression?

A) 1 B) 2 C) 4 D) 8

Simplify fully:

$$-5x(4 - 3x) + 2x^2$$

A) $2x^2 - 17x$ B) $2x^2 - 23x$ C) $17x^2 - 5x$ D) $17x^2 - 20x$

MOCK EQAO Multiple Choice

Which value of x satisfies the equation:

$$5 - 2x = 9$$

A) $x = -7$ B) $x = -2$ C) $x = 2$ D) $x = 3$

Simplify the following algebraic expression:

$$\frac{a^6 b^4}{a^2 b}$$

A) $\frac{a^3}{b^3}$ B) $\frac{a^4}{b^3}$ C) a^3b^3 D) a^4b^3

If x = 3, what is the value of $2x^2 + 5x$?

A) 21 B) 27 C) 33 D) 51

Suzette expands and simplifies the expression below.

$$2(3x^2 - 5x) + 4x(7 + x)$$

Which expression is equivalent to the one above?

A) $6x^2 + 22x$ B) $10x^2 + 18x$ C) $10x^2 - 38x$ D) $28x^2$

With 12.00$, Samuel and a friend are buying lunch from the menu below.

Menu Tax Included					
Soups & Salads		Sandwiches		Beverages	
Tomato Soup	1.95$	Ham & Cheese	4.65$	Soft Drink	1.35$
Green Salad	2.25$	Turkey	5.15$	Tea/Coffee	0.99$
		Hamburger	3.45$	Juice	1.75$

Which of the following orders could they buy with their 12.00$

A) Two soft drinks and two turkey sandwiches
B) One tomato soup, one tea and two ham & cheese sandwiches
C) One tomato soup, one juice, two green salads and one hamburger
D) One soft drink, one tea, one turkey sandwich and one ham & cheese sandwich

MOCK EQAO Multiple Choice

Amandine receives 10,000$.

Amandine keeps half her money and gives the rest to Bernadette.

Bernadette keeps half her money and gives the rest to Claude.

Claude keeps half his money and gives the rest to Danielle.

Danielle keeps half her money and the rest to Eugenie.

Which expression shows the dollar amount of money that Eugenie receives from Danielle?

A) $10000 \div 2^4$ B) $5000 \text{ x } \frac{1}{2} \text{ x } \frac{1}{2}$ C) $10000 \div \frac{1}{2} \div \frac{1}{2} \div \frac{1}{2} \div \frac{1}{2}$ D) $2500 \div 2$

Chloe works as a translator. She charges 21₵ for each word she translates.

Which expression should Chloe use to calculate her charge, in dollars, for translating a document with *n* words?

A) $\frac{21\ x\ n}{100}$ \$ B) $\frac{100}{21\ x\ n}$ \$ C) $\frac{n}{21\ x\ 100}$ \$ D) $\frac{21\ x\ 100}{n}$ \$

MOCK EQAO Open Response

Alexandre works part-time at a clothing store. He is paid an hourly rate of 10.25$/hr and also earns a commission of 3.5% of his total weekly sales.

Alexandre works at the store 12 hours a week.

If Alexandre's goal is to earn 150$ every week, what do his total weekly sales need to be?

Show your work.

MAJOR UNIT TASK

Accounting Mishap Part One:

You are an accountant and you must balance the books. A colleague of yours has submitted some bills to your department but someone spilt coffee of the only existing copy of the manifest.

Description	**Deposits**	**Withdrawals**	**Balance**
Opening Balance			6349,21
Sold calculators	43,91		
Bought Mini-iPads			
Bought 13 new desks		762,12	
Paid for license		54,21	
Closing Balance			4140,61

Required Terminology:

A deposit is an accounting term which indicates a credit such that the amount of money going into the account increases. A withdrawal is an accounting term which indicates a removal of funds from the account. In other words, money is leaving the account.

Using mathematical language and proper algebraic techniques, **find the missing expense.**

Accounting Mishap Part Two:

Your colleague alerts you that two of the new desks were damaged upon arrival and that you are to be refunded for the value of two desks.

Using mathematical language and proper algebraic techniques, **determine the value of each desk**.

Re-represent

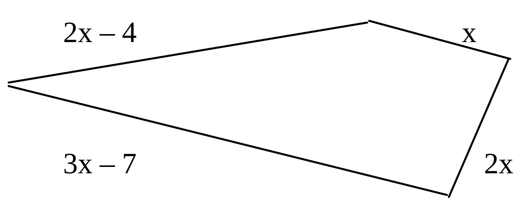
Using algeo-tiles, create an algebraic expression for :

$$3x^2 - 5x + 4$$

such that you must add or subtract tiles to obtain the above result. For example, to say 4x -1, draw 6x+4 and -2x-5.

Lost in Orléans

While walking to school, Amélie took a wrong turn. Her father tracked her down using the GPS locator in her phone to guide her back home. The diagram below is the path she took. The GPS states that she walked 5.2 km.



Using mathematical language and proper algebraic techniques, **determine the length of each pathway**.

Task/ Expectation	Level R	Level 1	Level 2	Level 3	Level 4
Simplify numerical and polynomial expressions in one variable, and solve simple first-degree equations.	Does not simplify numerical and polynomial expressions in one variable, and solve simple first-degree equations.	Struggles to simplify numerical and polynomial expressions in one variable, and solve simple first-degree equations.	Sometimes simplifies numerical and polynomial expressions in one variable, and solve simple first-degree equations.	Usually simplifies numerical and polynomial expressions in one variable, and solve simple first-degree equations.	Almost always simplifies numerical and polynomial expressions in one variable, and solve simple first-degree equations.

UNIT REVIEW

Evaluate the following:

$-15 - 12$: ________________________

$(-3)(6)$: ________________________

$-14.9 + 12.3$: ________________________

$\frac{-16}{2}$: ________________________

$(4)^3$: ________________________

$(\frac{-3}{4})^2$: ________________________

$(5x)(2x)$: ________________________

$(7x^2)(9x)$: ________________________

Solve for the unknown variable. Be sure to check.

$x + 17 = 52$

$6x = -72$

$f - 12 = 64$

$\frac{x}{3} = -5$

DID YOU CHECK YOUR ANSWERS?

Simplify the following.

$4(x-11)$

$2x(3x^2 - 4x - 1)$

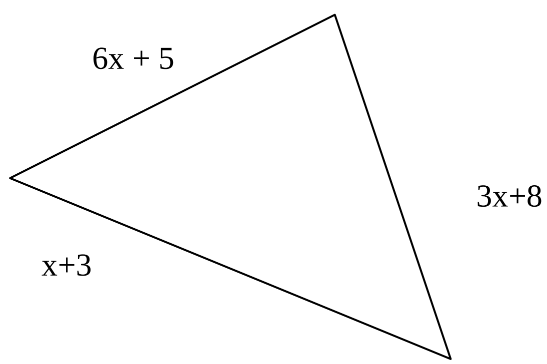
Simplify the following.

$3x – 7 + 2x –12$

$5x – 9(2x + 7)$

$2(3x-5) + 7(2x-8)$

Design a formula that models the perimeter of the following triangle.



If the Perimeter = 56 cm, what is the length of each side?

Evaluate the volume of a sphere with a radius of r = 12.4 cm.

$$V = \frac{4\pi r^3}{3}$$

Solve for the unknown variable.

$-15 + 5m = 20$

$\frac{x}{5} - 21 = 4$

DID YOU CHECK YOUR ANSWER?

During a FIFA tournament, four German players run onto a soccer pitch. A few minutes later, three Swedish players run onto the pitch. Another minute passes and a German player runs on followed by two Swedish players and then two German players.

Write two complete “let” statements to assign variables to each of the potential player nationalities arriving on the soccer field.

Your visually impaired friend asks you to provide an un-simplified algebraic statement using the variables you’ve chosen of the players arriving on the soccer field?

In the form of a script or sentence, how should you summarize the number of players on the pitch such that it is the most efficient manner?

Using algeo-tiles draw a representation of the following:

$2x – 5$

$2x^2 – 4x + 1$